A Remembrance and What Happened Next

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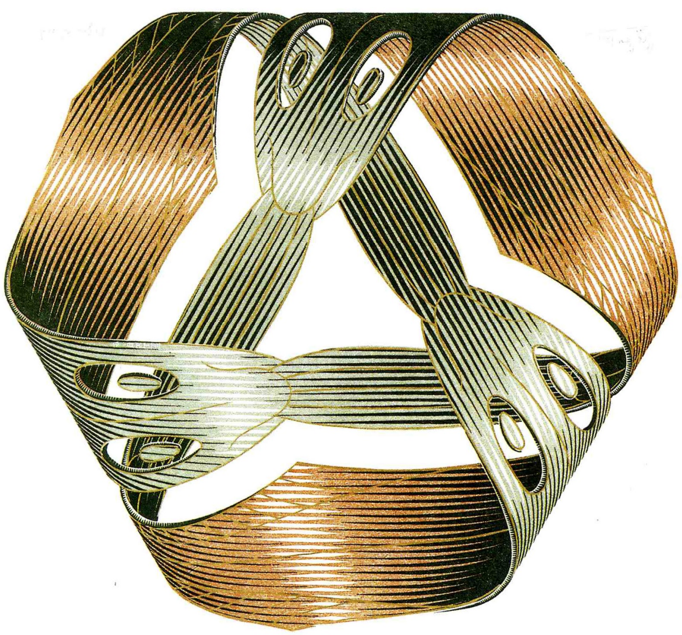
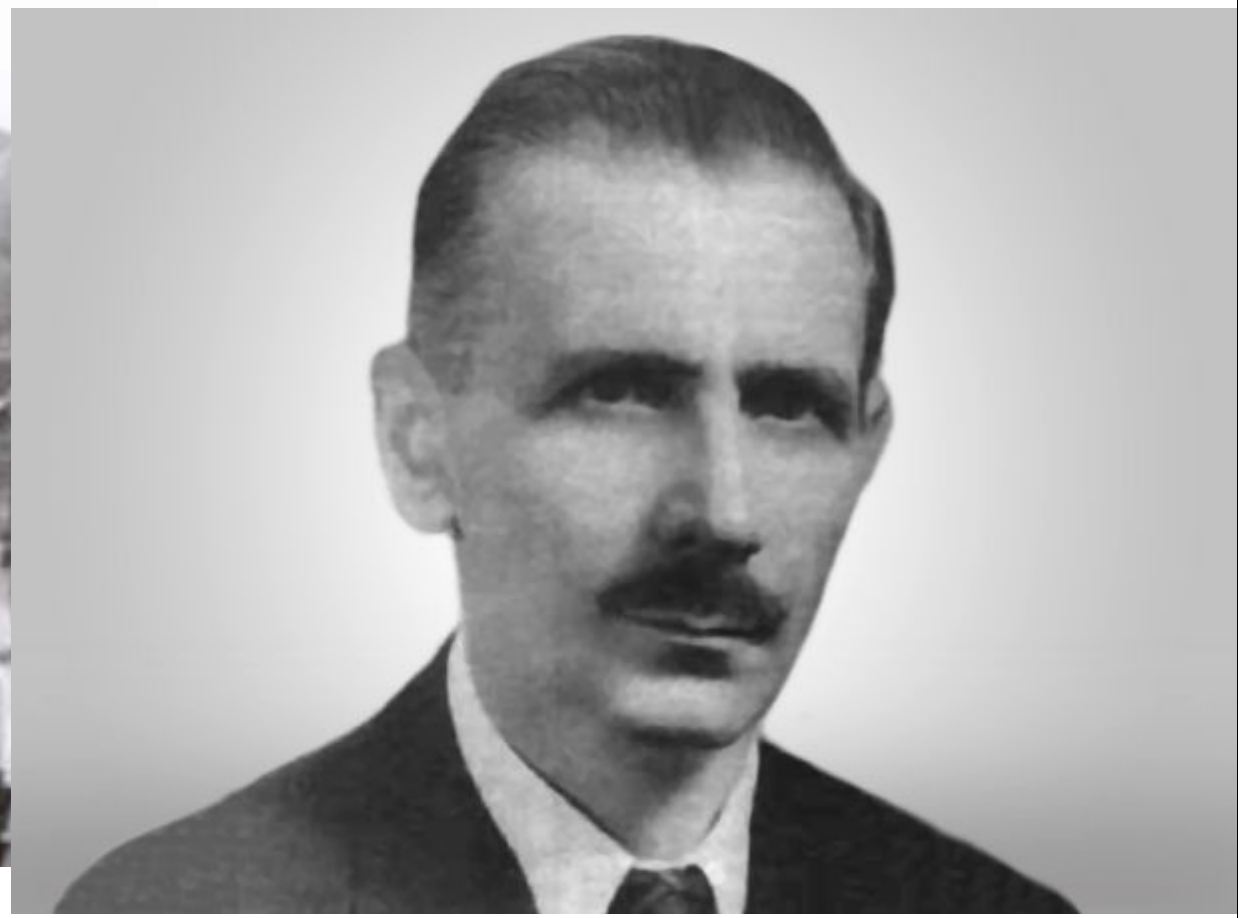


Many of the photgraphs
of Old Fine Hall are
from the collection of
Jay Goldman:

http://www.math.sunysb.edu/~tony/album/finehall0.html



Papakryiakopoulos was a daily figure in the Princeton Common Room

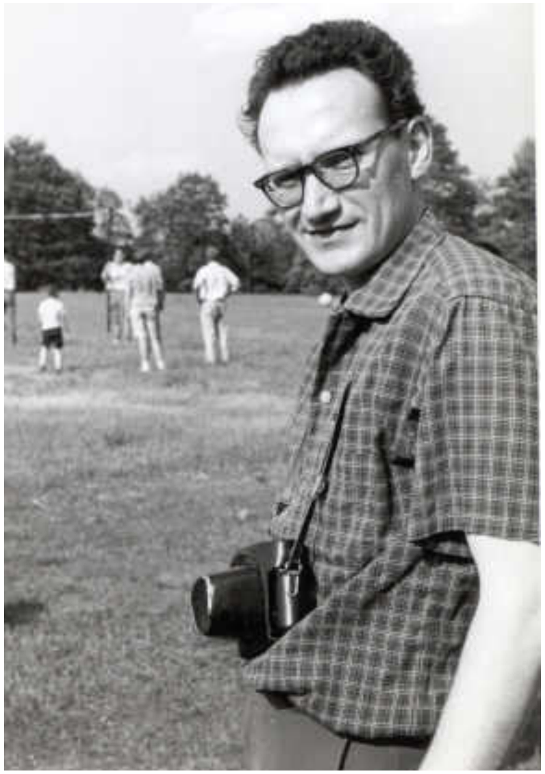
He would often have the newspaper up and I recall it was a Greek newspaper.

Not Papa back there, but
this gives a feel of the
room.

Ralph Fox
was another
presence in the
common room.

Here is a shot of
Fox and Hale Trotter
in conversation.

I entered Princeton as a graduate student in 1966.



Eventually, my advisor was William Browder.

My interests were in the interaction of low dimensional and high dimensional topology, and I learned as much knot theory as I could from Fox and worked in his small seminar where we reported on classic papers.

(A recent picture of Deborah Goldsmith)

Deborah Goldsmith was a student of Fox and we worked together in that seminar.

Fox was very good at talking with students.

We were shy about approaching Papakriakopoulos.

But at some point, I think it was in 1969, Papa invited Deborah and myself to dinner.
We joined him in a Princeton restaurant and he talked about coming to the USA and working in Princeton.
He encourged us to work hard at our problems.
And he said
'I know you have to publish many papers.
It is important for you to do that. It is not so necessary for me any more.'

Looking back on his statement it has many meanings. Young people need to publish, but also he had accomplished his main thoerems and had the freedom to work fully on the Poincare Conjecture.

Meeting Papa made a lasting impression of his honesty and dedication in working on hard mathematical problems.

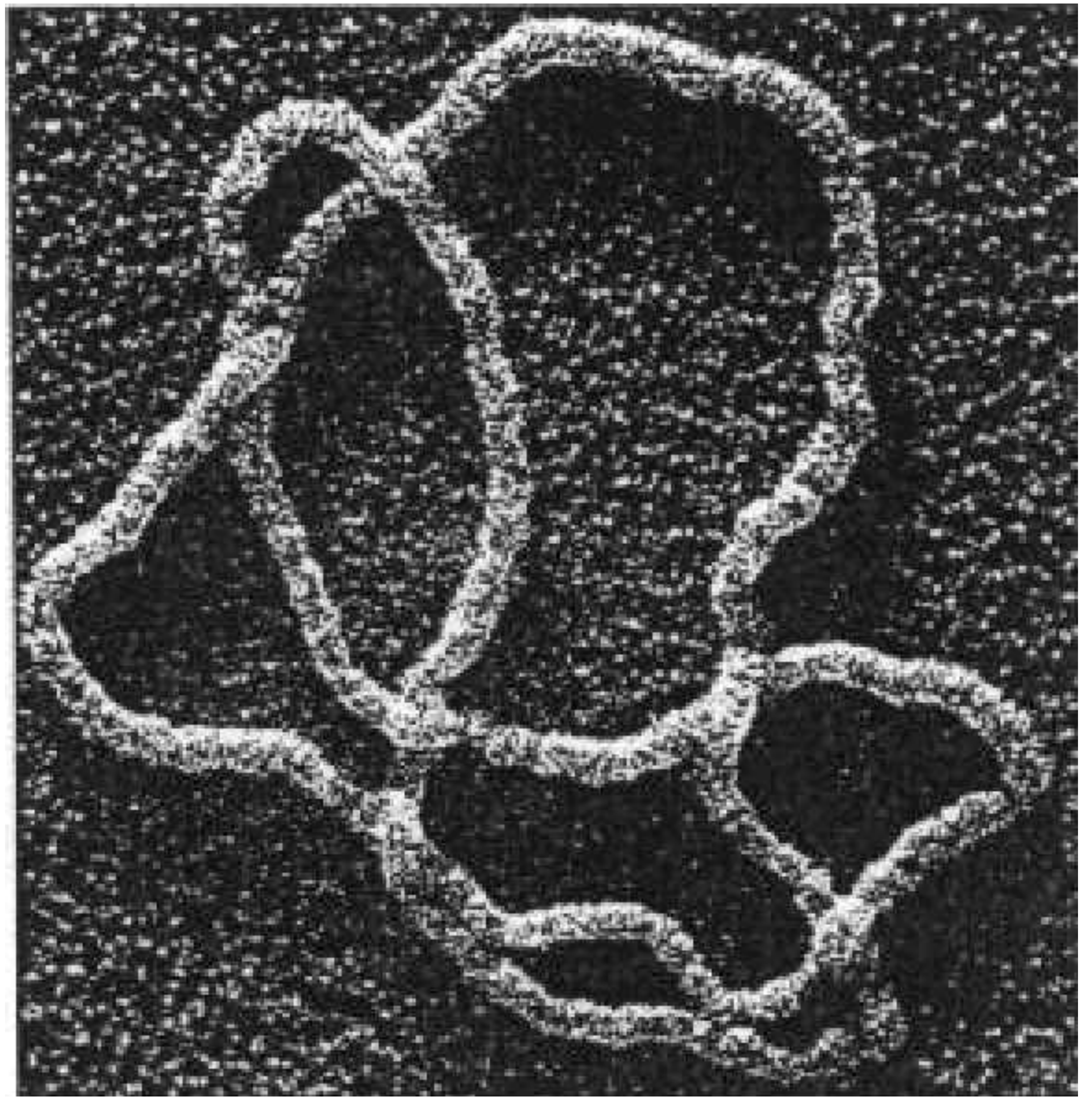
Old and new points of view recombined in low-dimensional topology in the years after that. The rest of this talk is intended to give an impression about this change.

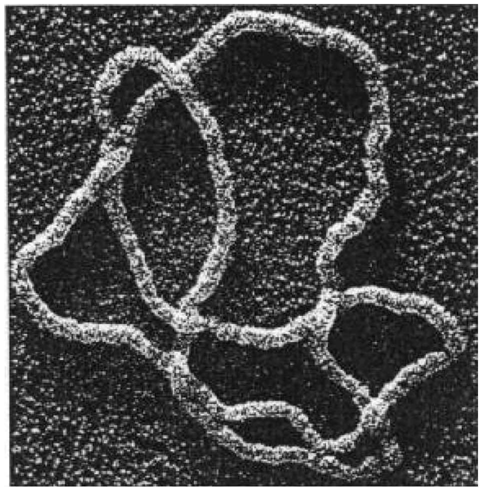
One thing that does not change is the ever-present possibility that new techniques will solve the problems that seemed so challenging in an older framework.

Changing Viewpoints

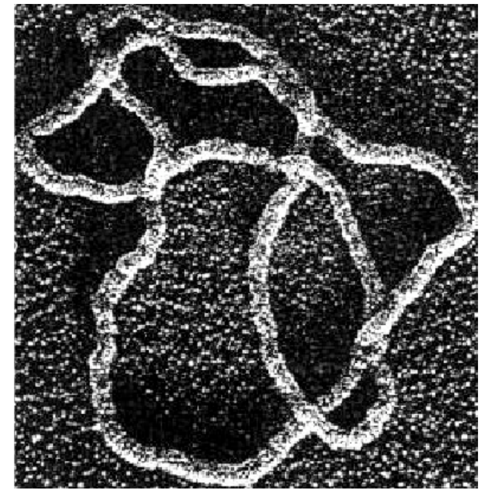
Three Dimensional Topology and Natural Science

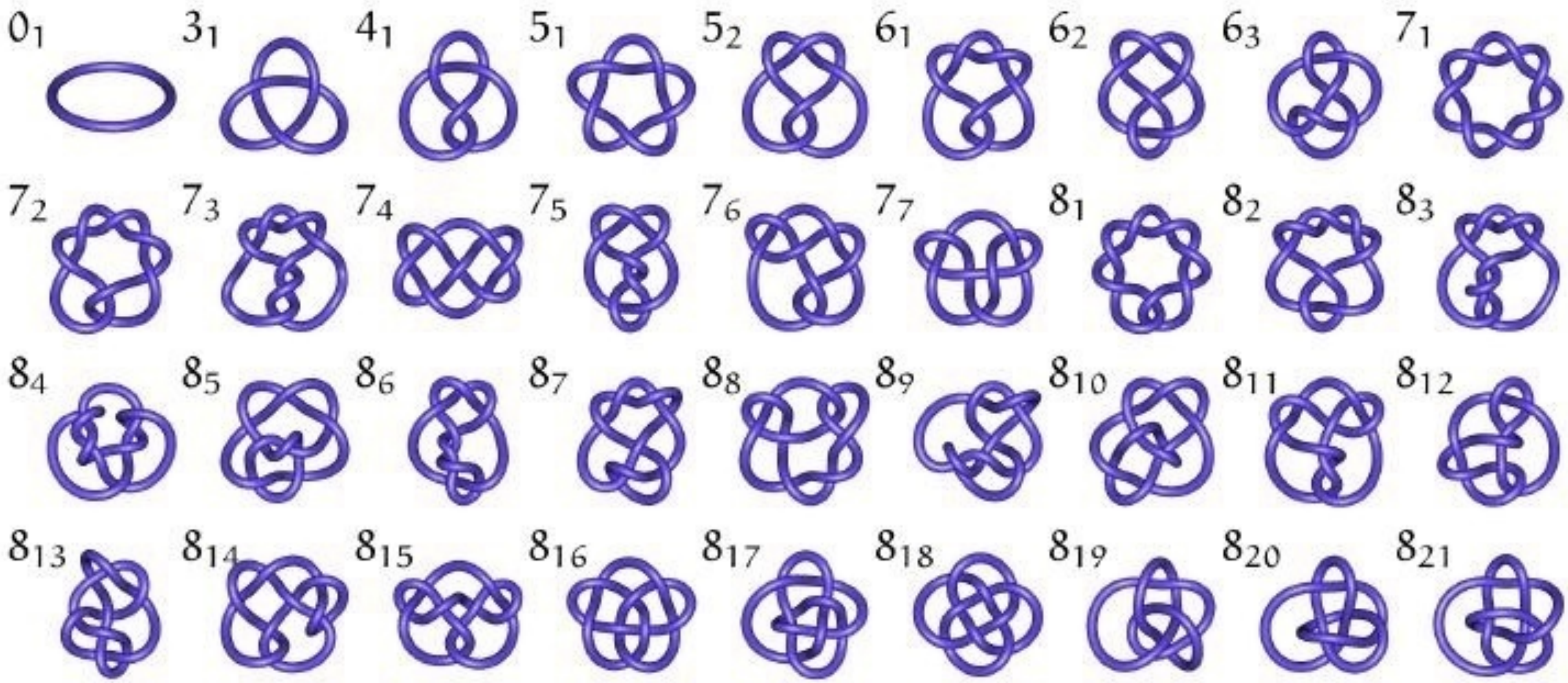
Knotted DNA - Electron Micrograph, Protein Coated DNA Molecule





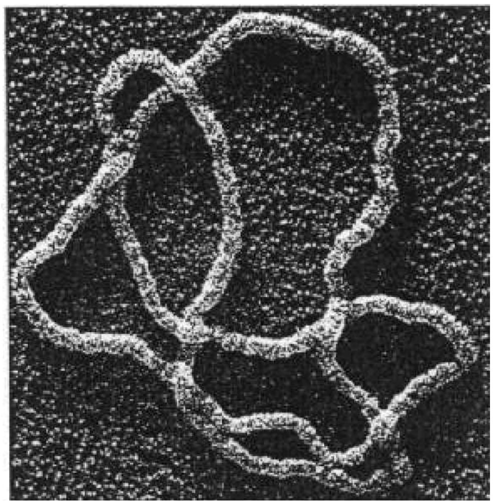
rotate





0_1 3_1 4_1 5_1 5_2 6_1 6_2 6_3 7_1
7_2 7_3 7_4 7_5 7_6 7_7 8_1 8_2 8_3
8_4 8_5 8_6 8_7 8_8 8_9 8_{10} 8_{11} 8_{12}
8_{13} 8_{14} 8_{15} 8_{16} 8_{17} 8_{18} 8_{19} 8_{20} 8_{21}

DNA Knotting and Recombination



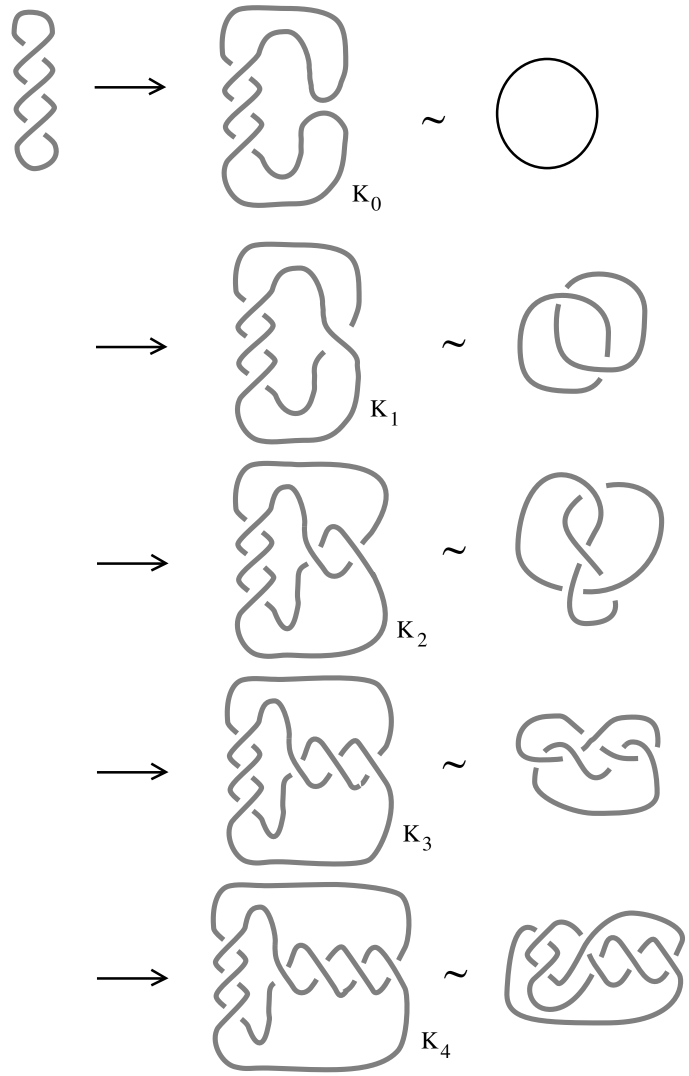
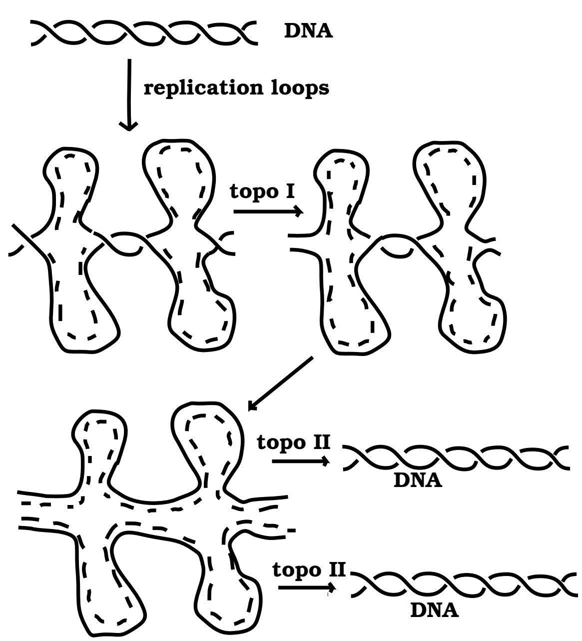


Figure 28 - Processive Recombination with $S = [-1/3]$.

DNA is a Self-Replicating Form



$$DNA = \langle W|C \rangle$$

$$\langle W| = \langle ...TTAGAATAGGTACGCG...|$$

$$|C \rangle = |...AATCTTATCCATGCGC... \rangle .$$

$$\langle W| + E \longrightarrow \langle W|C \rangle = DNA$$

$$E + |C \rangle \longrightarrow \langle W|C \rangle = DNA$$

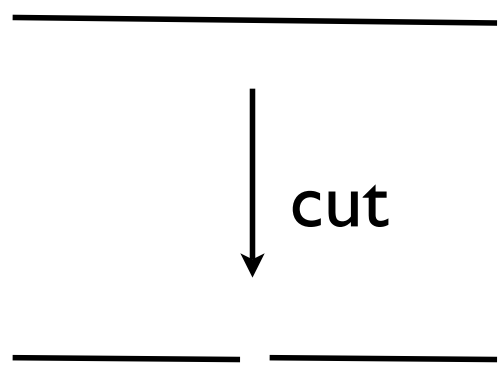
$$\langle W|C \rangle \longrightarrow \langle W| + E + |C \rangle = \langle W|C \rangle \langle W|C \rangle$$

Self Replication Schematic

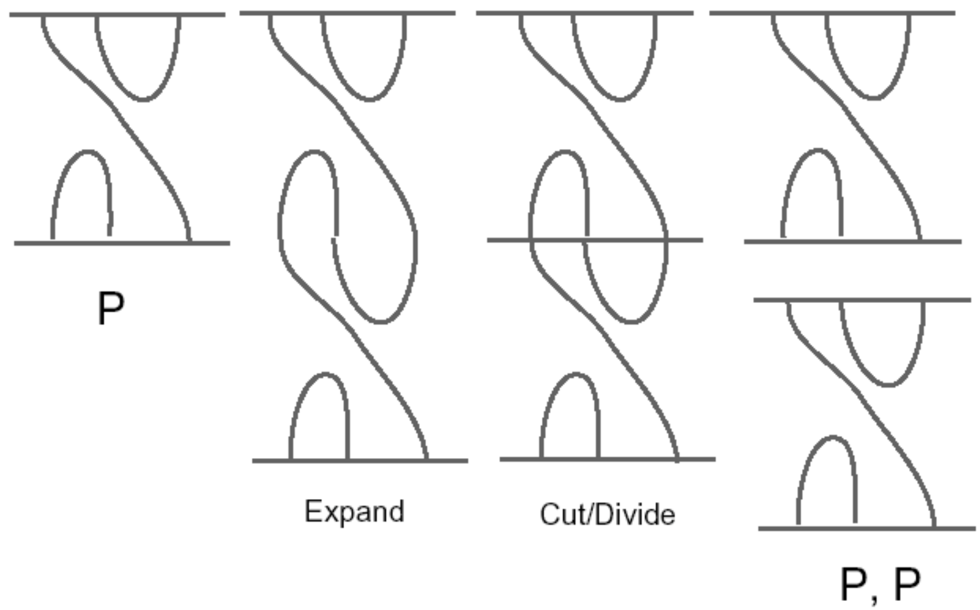
DNA = <Watson|Crick>

E = Environment

Simplest Replication

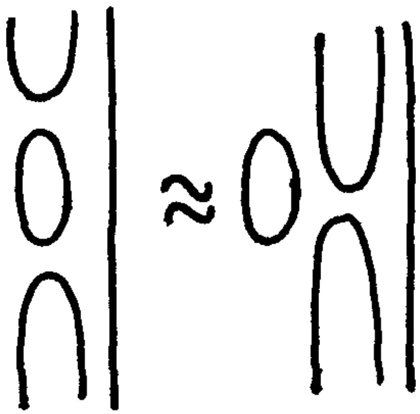


Topological Replication

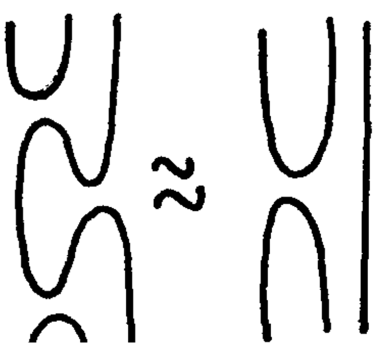


Temperley Lieb Algebra

Id U_1 U_2 $U_1 U_2$ $U_2 U_1$

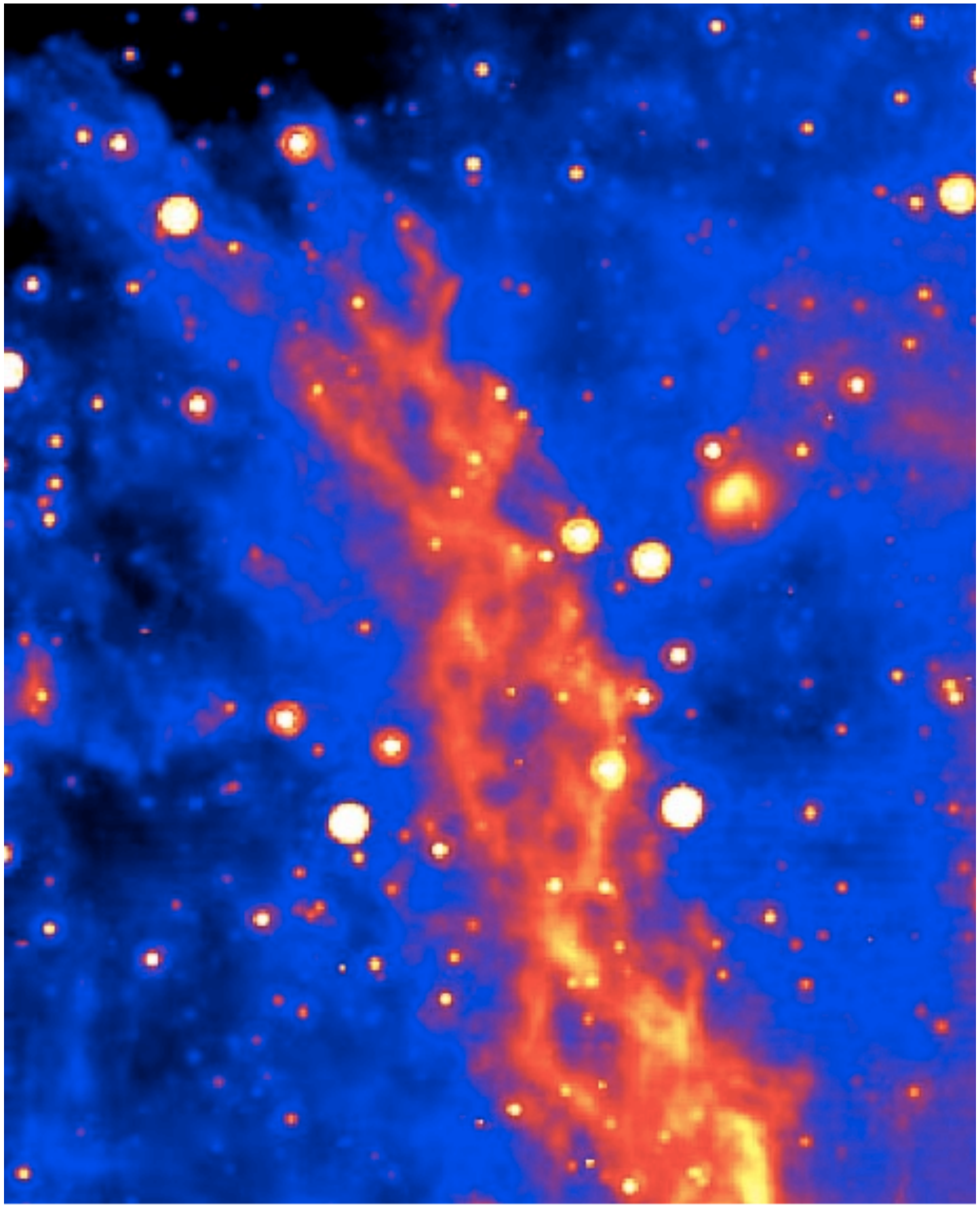


$$U_k^2 = dU_k$$



$$U_k U_{k+1} U_k = U_k$$

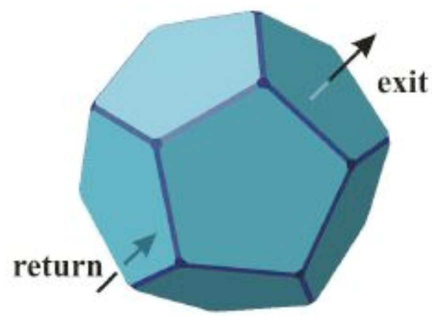
And $U_i \, U_j = U_j \, U_i$ when $|i - j| > 2$.

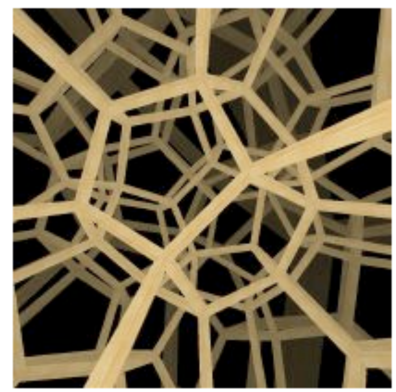


The DNA nebula
is an 80 light year
long formation
lying near the
enormous black hole
at the center of our
Milky Way galaxy.

http://news.nationalgeographic.com/news/2006/03/0317_060317_dna_nebula.html

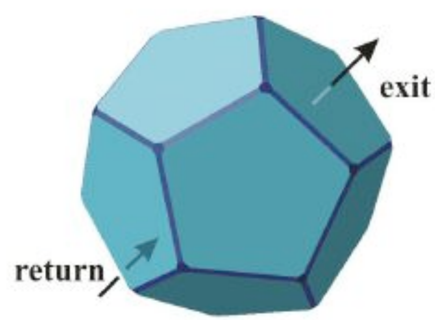
Is the Geometric Universe a Poincare Dodecahedral Space?

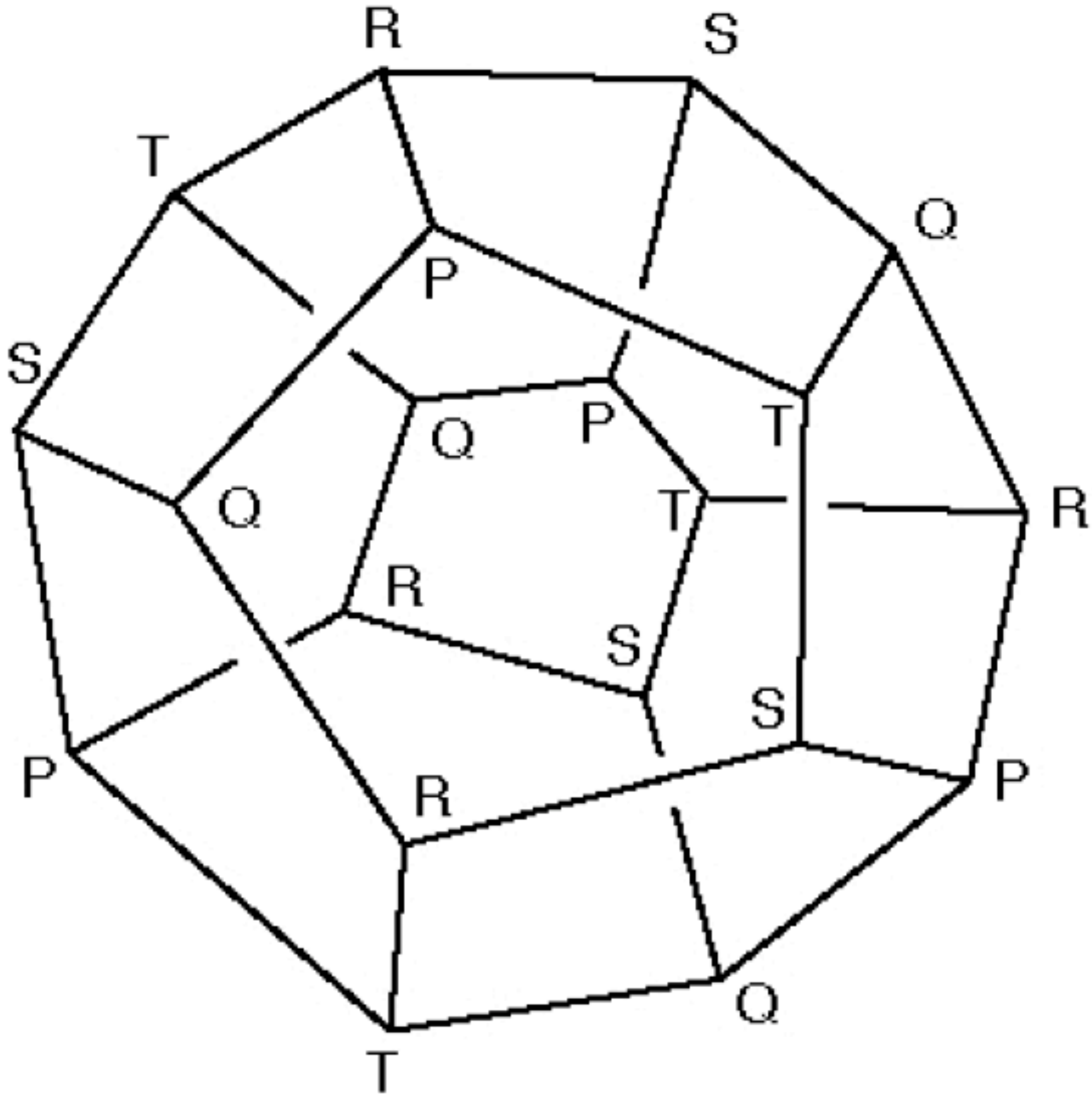




A franco-american team of cosmologists [1] led by J.-P. Luminet, of the Laboratoire Univers et Théories (LUTH) at the Paris Observatory, has proposed an explanation for a surprising detail observed in the Cosmic Microwave Background (CMB) recently mapped by the NASA satellite WMAP. According to the team, who published their study in the 9 October 2003 issue of Nature, an intriguing discrepancy in the temperature fluctuations in the afterglow of the big bang can be explained by a very specific global shape of space (a "topology"). The universe could be wrapped around, a little bit like a "soccer ball", the volume of which would represent only 80% of the observable universe! (figure 1) According to the leading cosmologist George Ellis, from Cape Town University (South Africa), who comments on this work in the "News & Views" section of the same issue: "If confirmed, it is a major discovery about the nature of the universe".

The Poincare Dodecahedral space is
obtained by identifying opposite
sides of a dodedahedron with
a twist.
The resulting space, if you were inside it,
would be something like the next slide.
Whenever you crossed a pentagonal face,
you would find yourself back in
the Dodecahedron.




R
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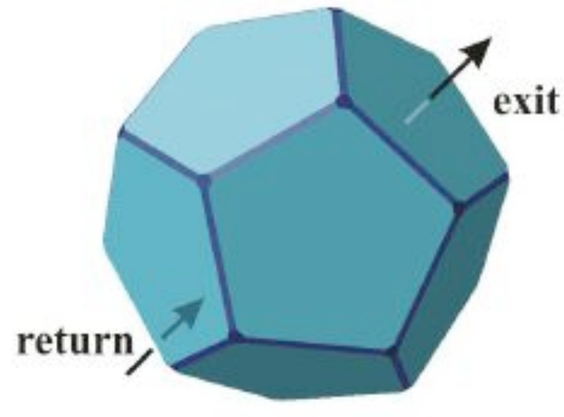
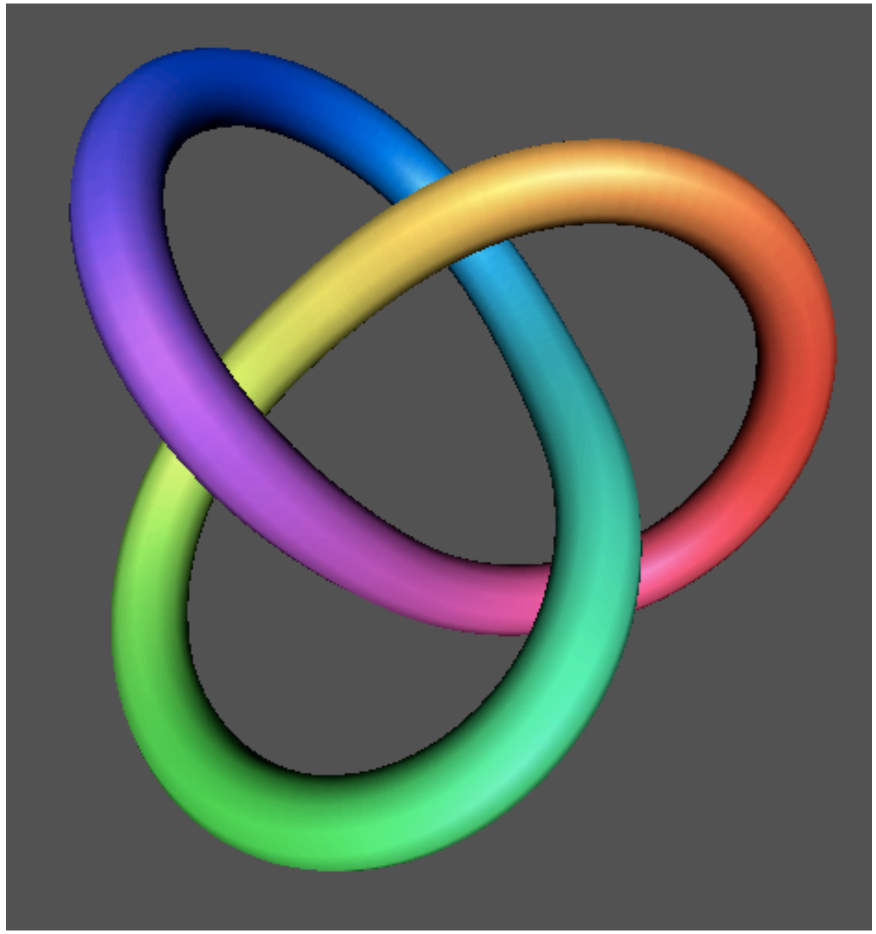
What Does This Have
to do with Knot Theory?



The dodecahedral Space M has
Axes of Symmetry:
five-fold, three-fold and two-fold.

The dodecahedral space M is the
5-fold cyclic branched covering
of the three-sphere, branched along the
trefoil knot.

M = Variety(x^2 + y^3 + z^5)
Intersected with S^5 in C^3.



So perhaps the trefoil knot is the key to the universe.

Creation and Dynamics of Knotted Vortices

Dustin Kleckner[1] & William T. M. Irvine[1]

[1]*James Franck Institute, Department of Physics, The University of Chicago, Chicago, Illi*

60637, USA

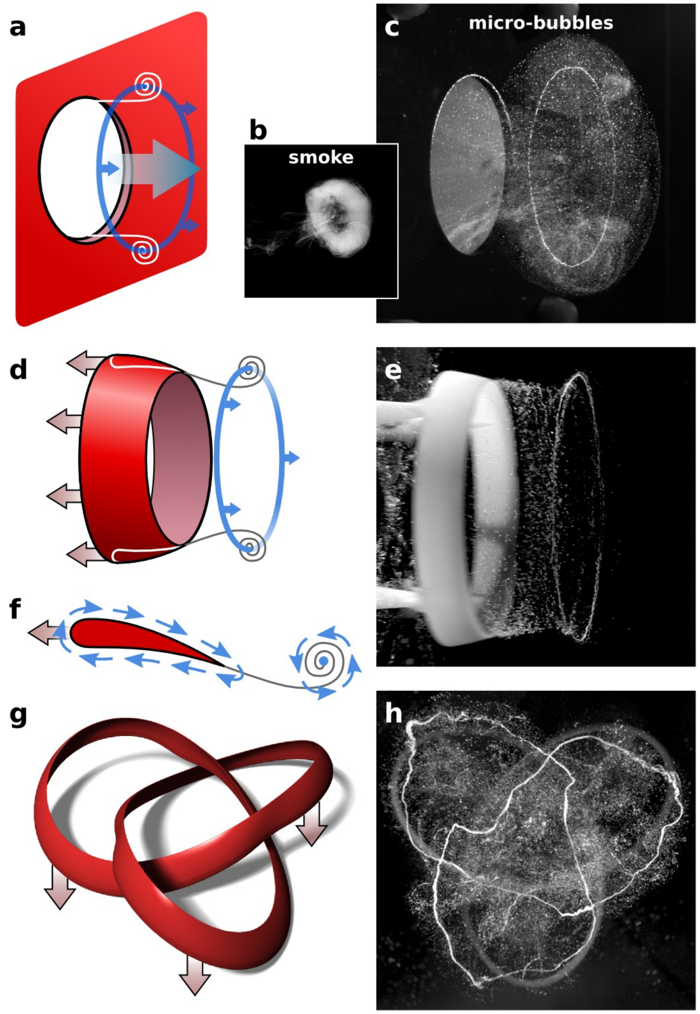
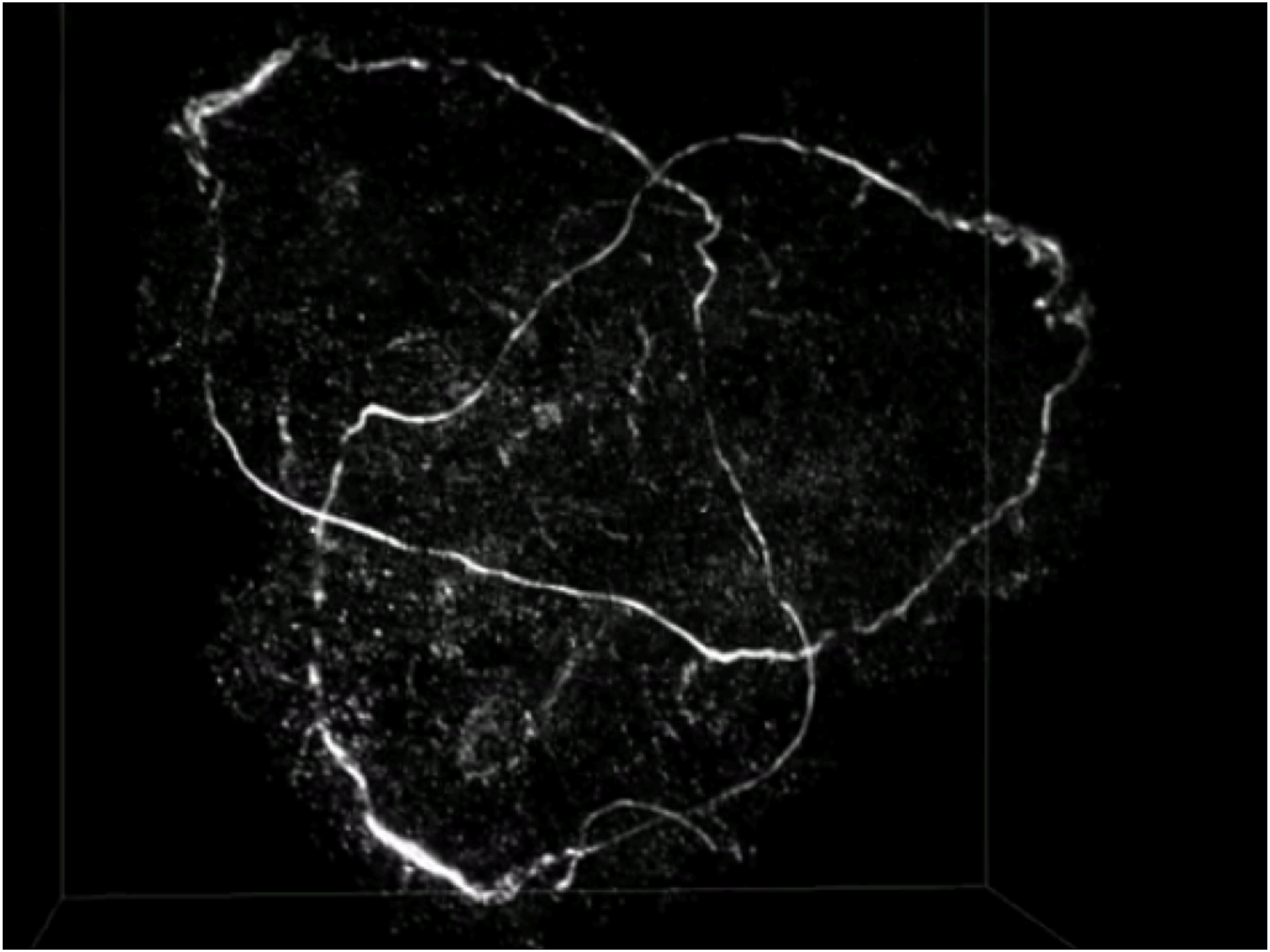


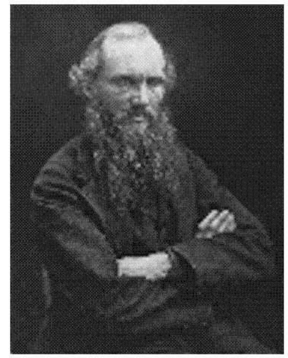
FIG. 1. The creation of vortices with designed shape and topology. **a,** The conventional method for generating a vortex ring, in which a burst of fluid is forced through an orifice. **b,** A vortex ring in air visualized with smoke. **c,** A vortex ring in water traced by a line of ultra-fine gas bubbles, which show finer core details than smoke or dye. **d-e,** A vortex ring can alternatively be generated as the starting vortex of a suddenly accelerated, specially designed wing. For a wing with the trailing edge angled inward, the starting vortex moves in the *opposite* of the direction of wing motion **f,** The starting vortex is a result of conservation of circulation – the bound circulation around a wing is balanced by the counter-rotating starting vortex. **g,** A rendering of a wing tied into a knot, used to generate a knotted vortex, shown in **h**.



Lord Kelvin's Vortex Atoms

`Idea of knotted strings as fundamental constituents of matter is old`

Lord Kelvin and the
1867 string revolution:



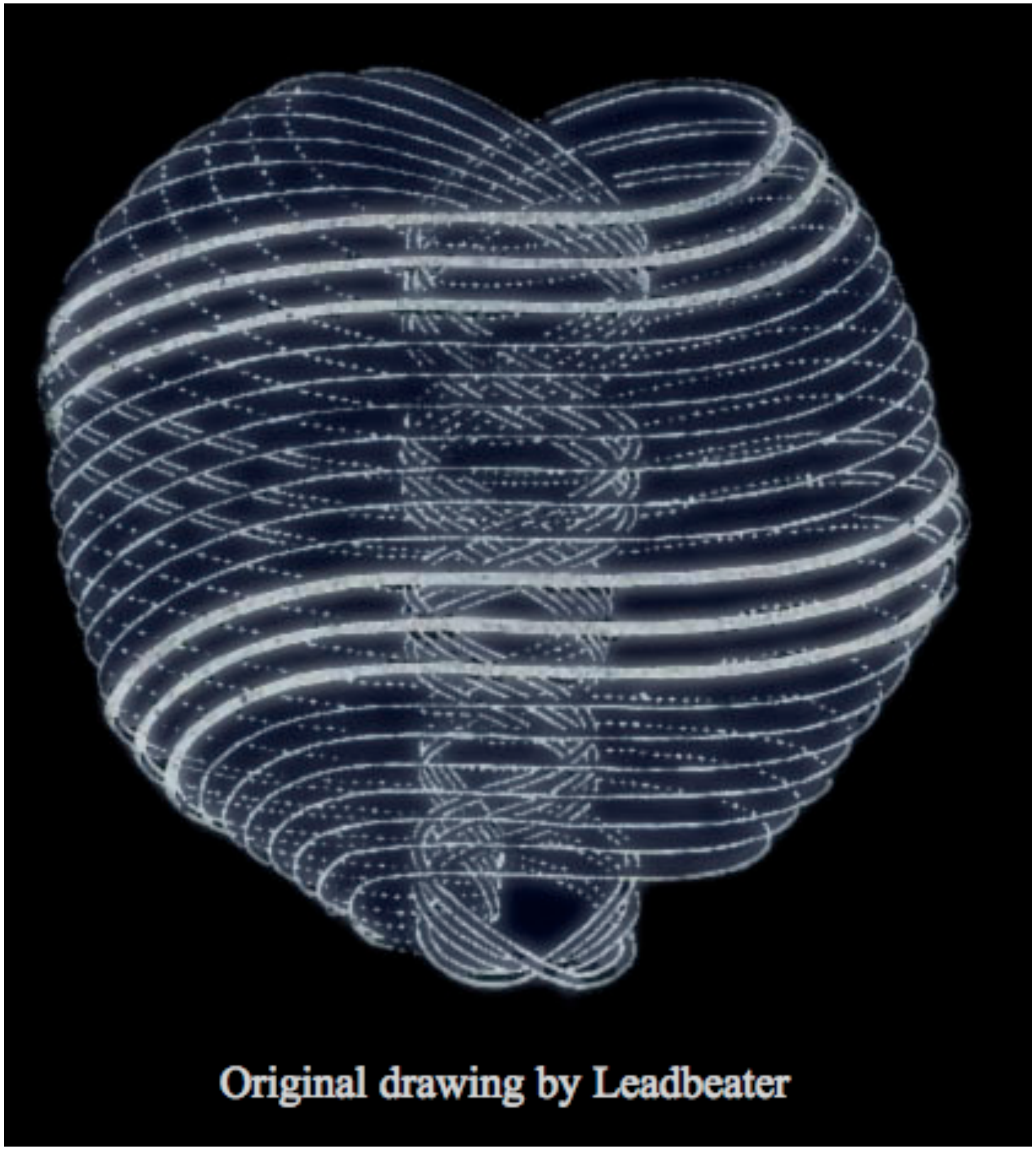
atoms are knotted tubes of aether

- topological stability of knots = stability of matter

- variety of knots = variety of chemical elements

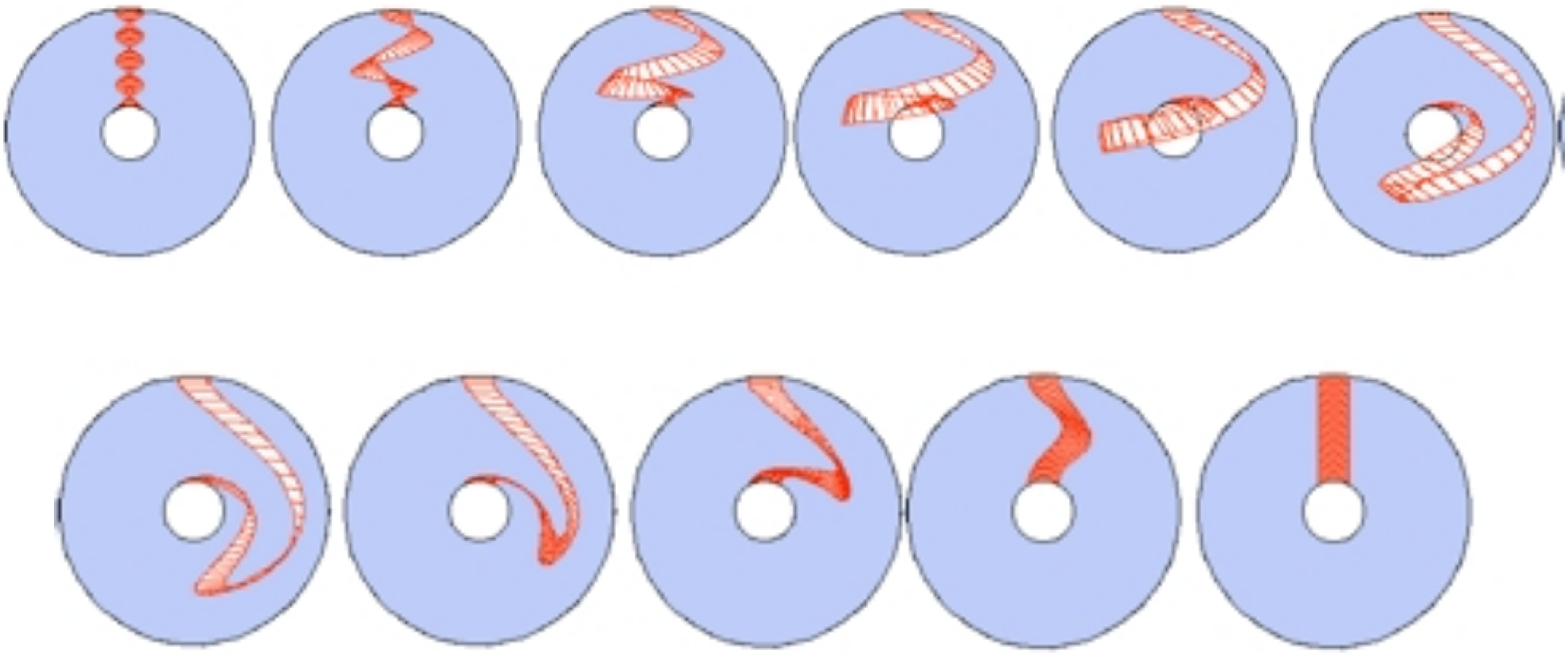
For decades considered as *the* theory of fundamental Matter

`Maxwell:` *Kelvin's theory satisfies more of the conditions than any atom hitherto considered*



From the same period as Kelvin, the “vortex atom” of the visionaries Besant and Leadbeater.

The Dirac String Trick



Quaternion Demonstrator

$ii = jj = kk = ijk = -1$

$ij = k$

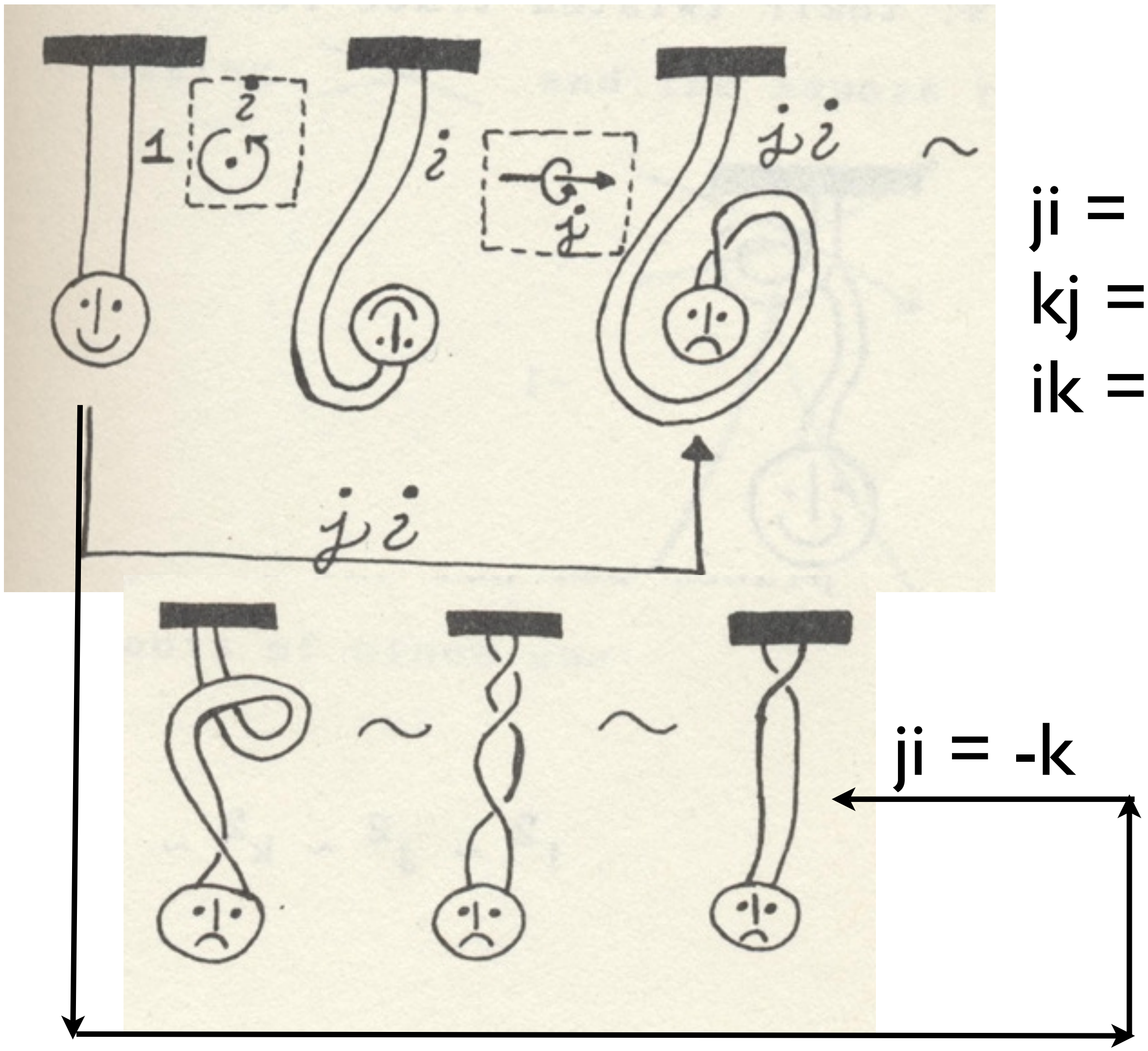
$jk = i$

$ki = j$

$ji = -k$

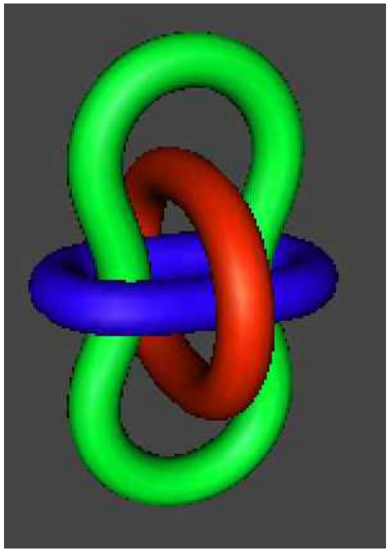
$kj = -i$

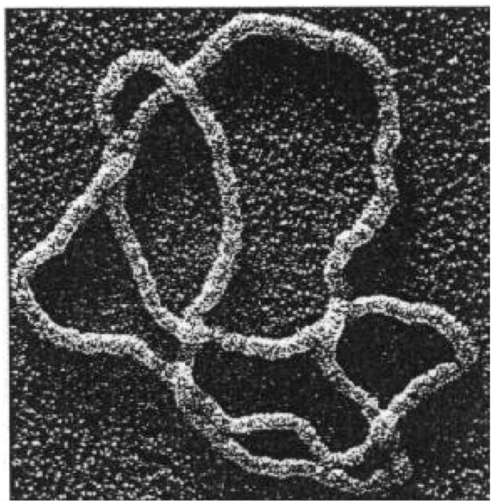
$ik = -j$




Air on the
Dirac Strings

Graphs, Diagrams and Reidemeister Moves





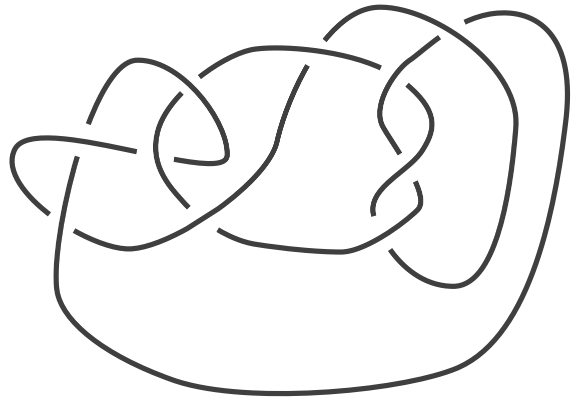


Figure 1 - A knot diagram.

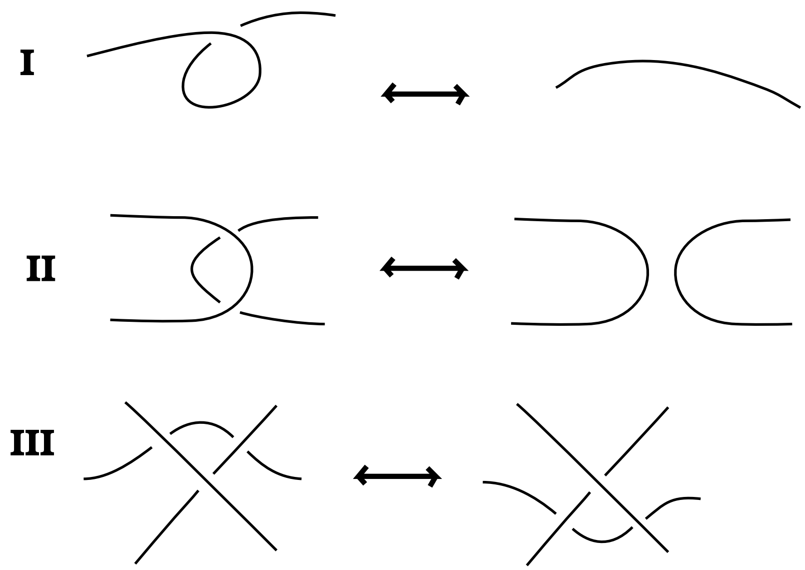
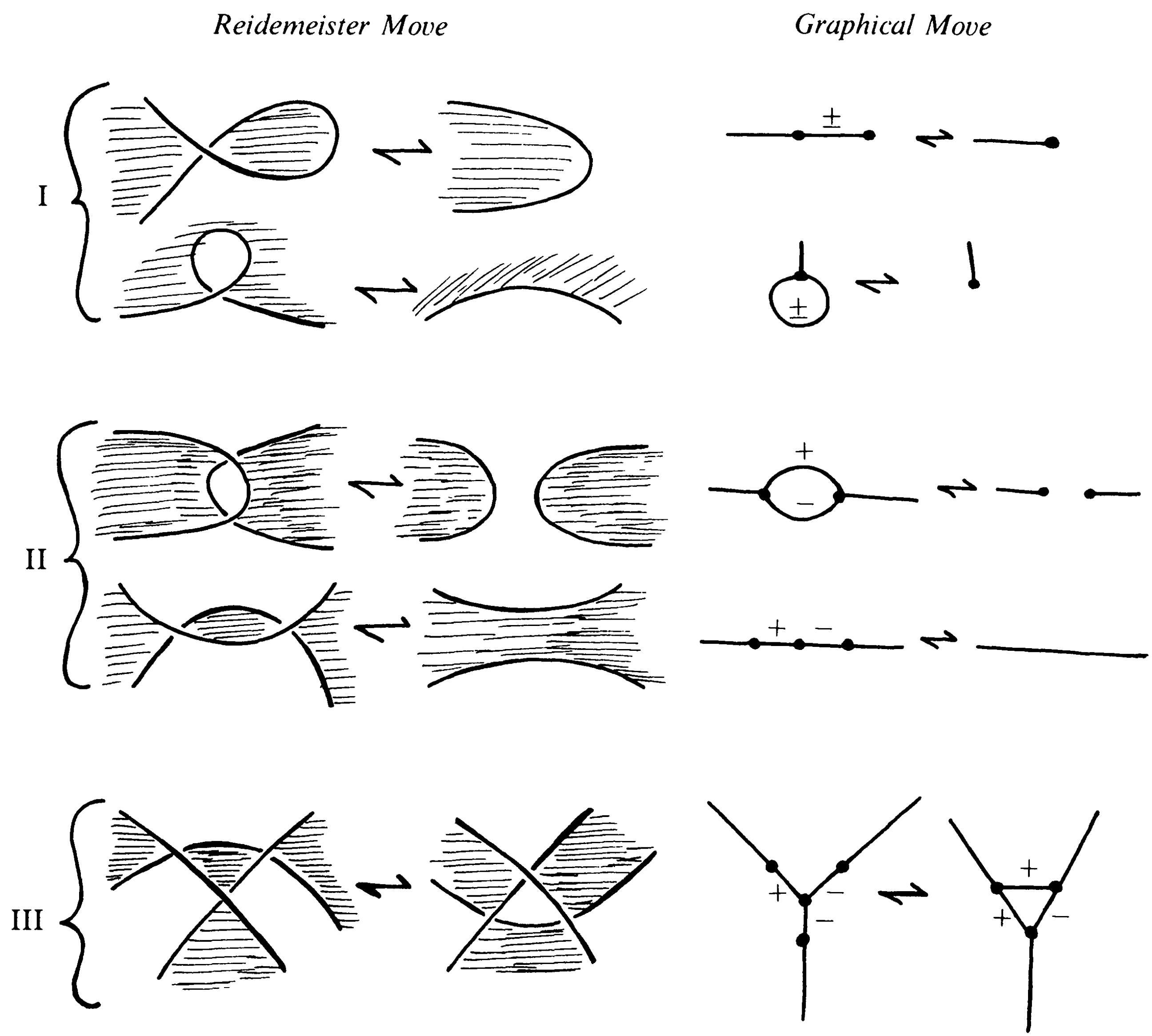
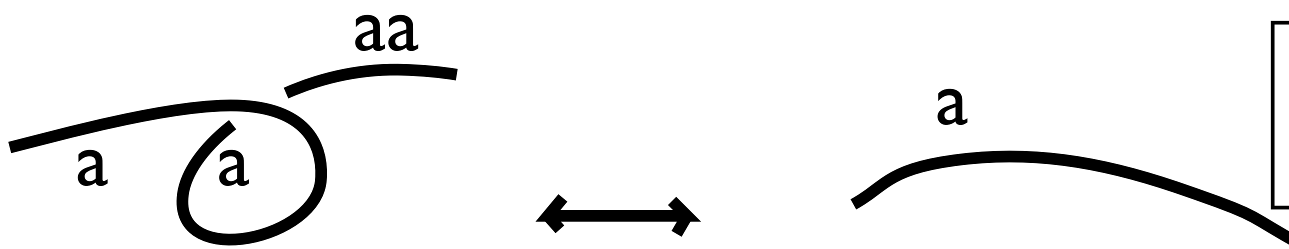


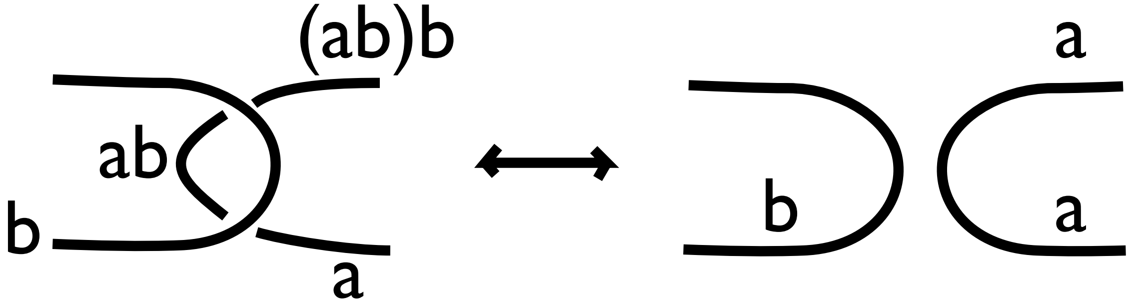
Figure 2 - The Reidemeister Moves.

Graphical Reidemeister Moves and the Electrical Analogy



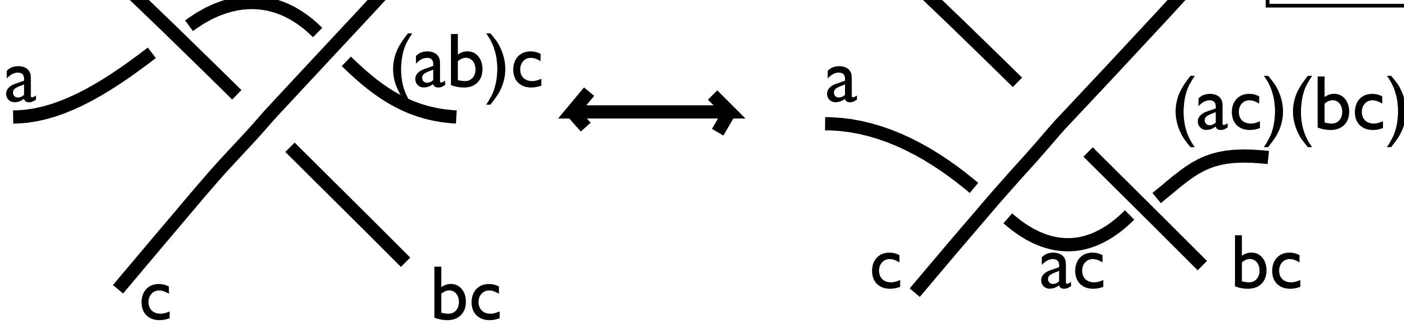


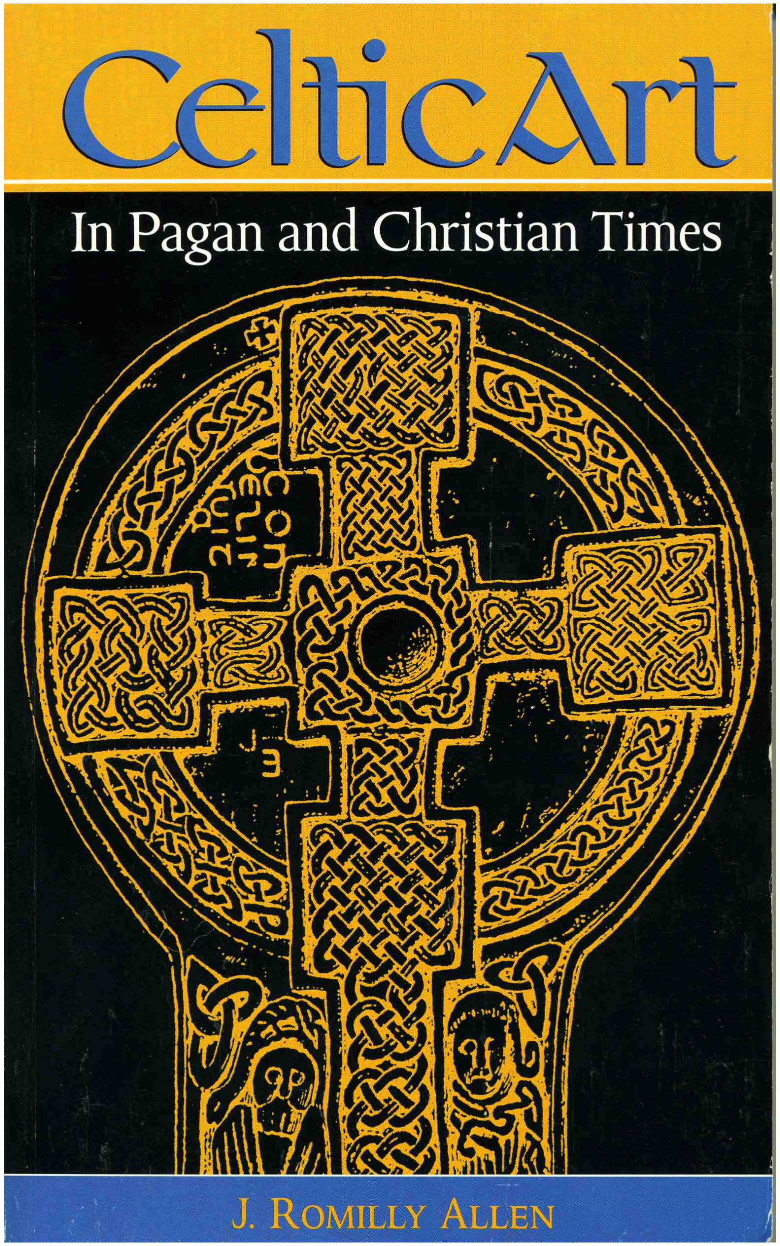
$aa = a$



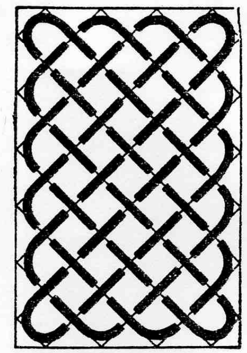
$(ab)b = a$

$(ab)c = (ac)(bc)$



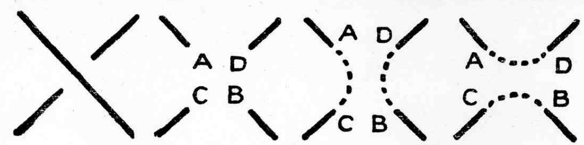


I now propose to explain how plaitwork is set out, and the method of making breaks in it. When it is required to fill in a rectangular panel with a plait the four sides of the panel are divided up into equal parts (except at the ends, where half a division is left), and the points thus found are joined, so as to form a network of diagonal lines. The plait is then drawn over these lines, in the manner shown on the accompanying diagram. The setting-out lines ought really to be double so as to define the width of the band composing the plait, but they are drawn single on the diagram in order to simplify the explanation.



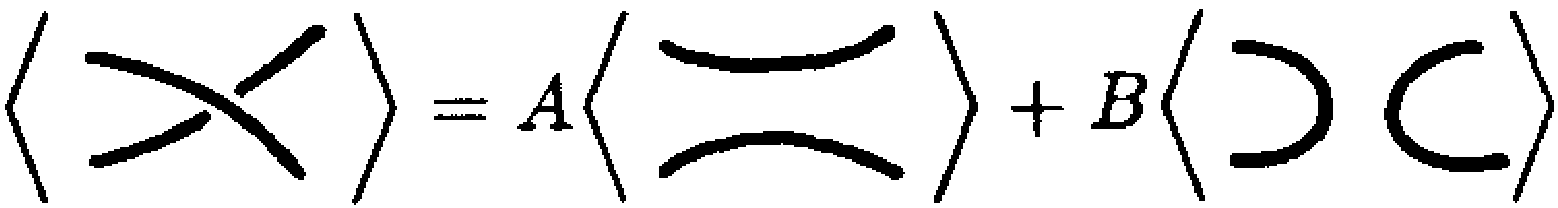
Regular plaitwork without any break

If now we desire to make a break in the plait any two of the cords are cut asunder at the point where they cross each other, leaving four loose ends A, B, C, D. To make a break the loose ends are joined together in pairs. This can be done in two ways only: (1) A can be joined to C and D to B, forming a vertical



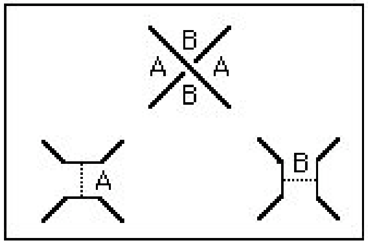
Method of making breaks in plaitwork

break; or (2) A can be joined to D and C to B, forming a horizontal break. The decorative effect of the plait is thus entirely altered by running two of the meshes


= A
+ B

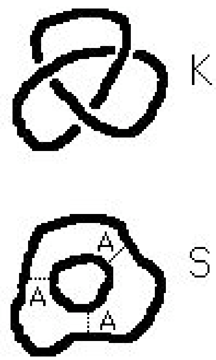
The Bracket Polynomial

(giving a state summation model for the Jones polynomial)

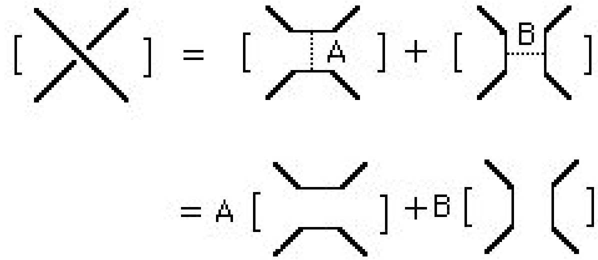


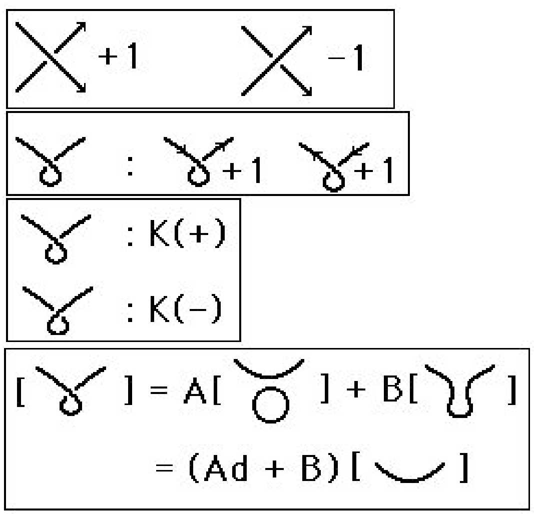
$||S|| = 2$

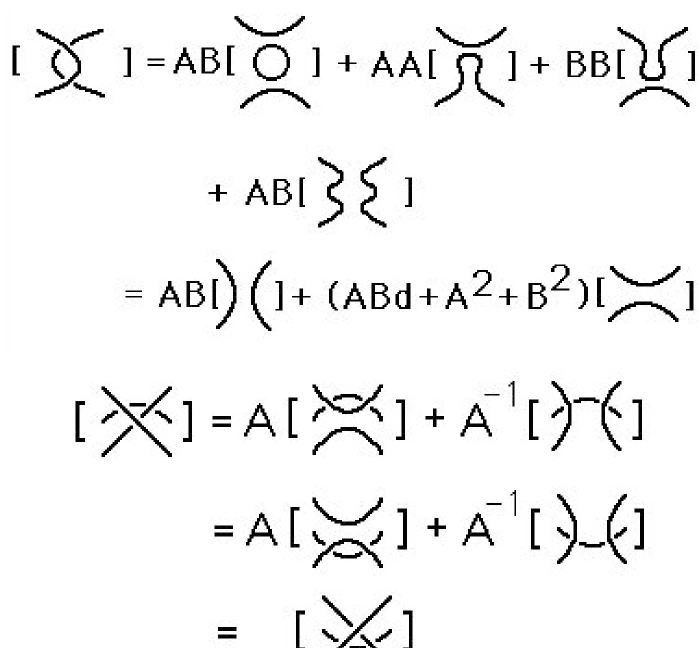
$[K|S] = A^3$

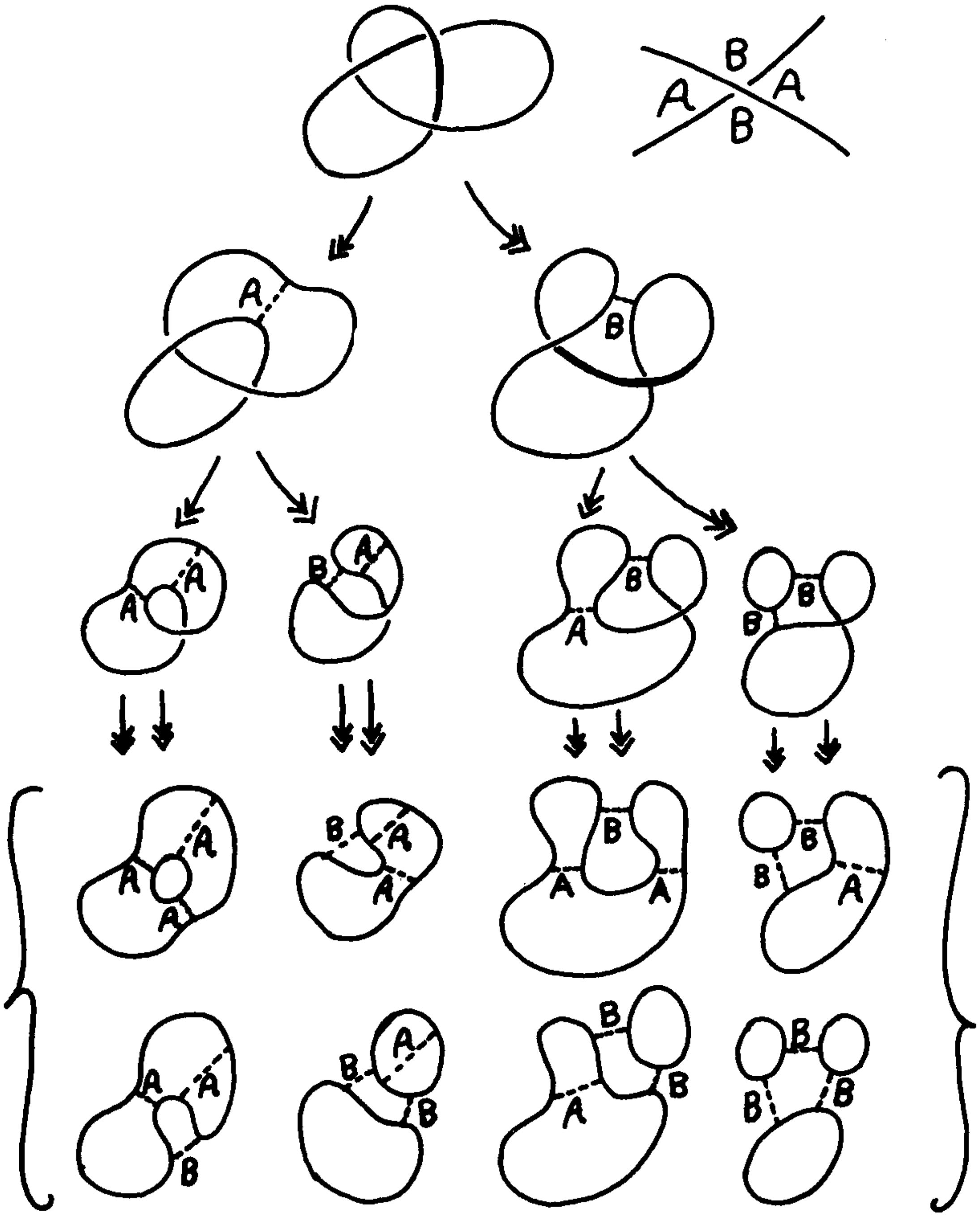


$$[K] = \sum_S [K|S] d^{||S||}$$







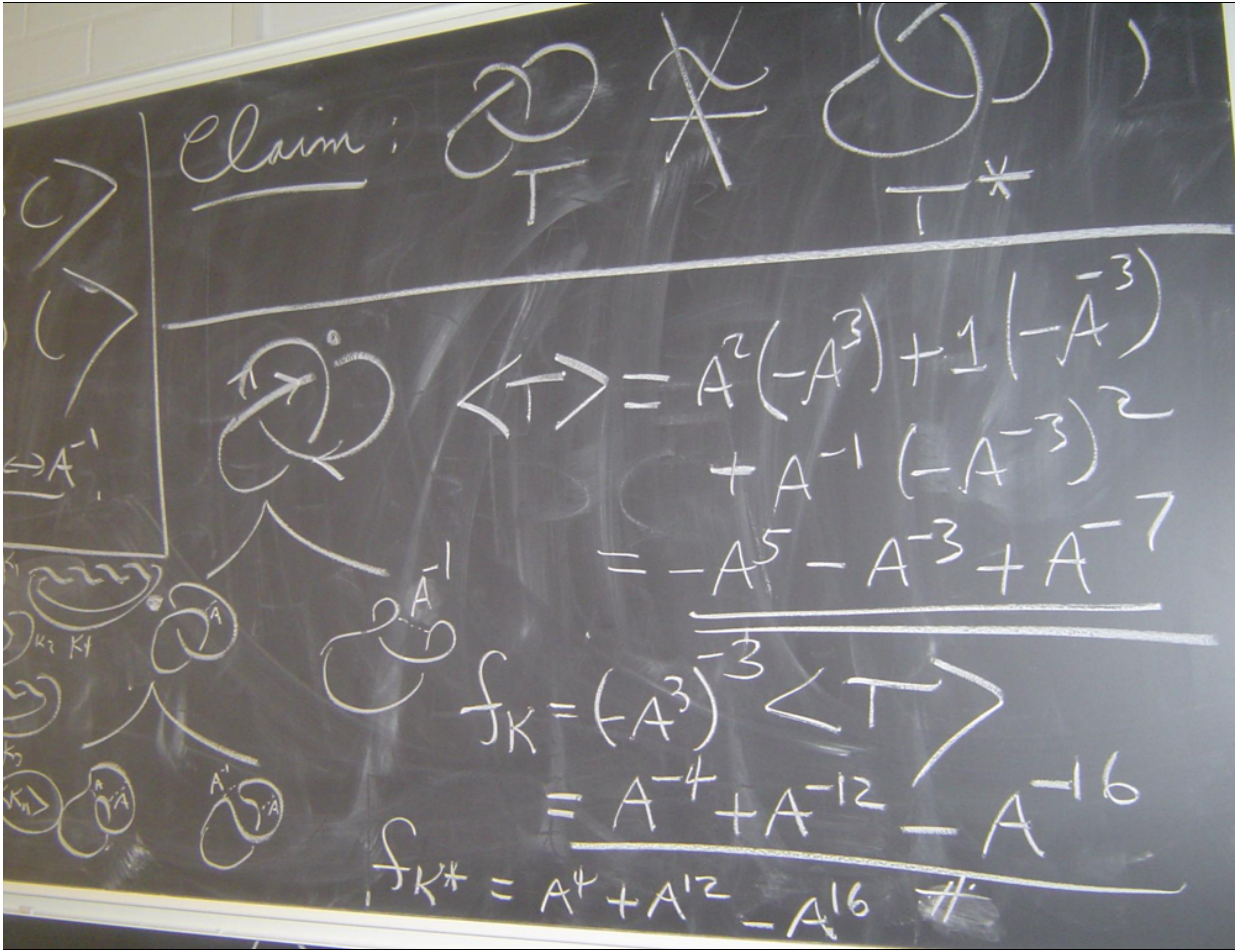

B
A A
B

Getting Invariance under all Reidemeister Moves

$$fK(A) = (-A^{3})^{(-w(K))}\langle K\rangle(A)$$

$w(K)$ = the sum of the crossing signs of K

Then $fK(A)$ is invariant under all three Reidemeister moves.



Claim: T ≠ T*
$\langle T \rangle = A^2(-A^3) + 1(-A^{-3})$
$+ A^{-1}(-A^{-3})^2$
$= -A^5 - A^{-3} + A^{-7}$
$f_K = (-A^3)^{-3} \langle T \rangle$
$= A^{-4} + A^{-12} - A^{-16}$
$f_{K^*} = A^4 + A^{12} - A^{16}$ #

$\Big\langle$  $\Big\rangle = A \Big\langle$  $\Big\rangle + A^{-1} \Big\langle$  $\Big\rangle$

L

$$= A(-A^3) + A^{-1}(-A^{-3})$$

$$\langle L \rangle = -A^4 - A^{-4}.$$

$\Big\langle$  $\Big\rangle = A \Big\langle$  $\Big\rangle + A^{-1} \Big\langle$  $\Big\rangle$

T

$$= A(-A^4 - A^{-4}) + A^{-1}(-A^{-3})^2$$

$$\langle T \rangle = -A^5 - A^{-3} + A^{-7}$$

$w(T) = 3$ (independent of the choice of orientation since T is a knot)

$$\therefore \mathcal{L}_T = (-A^3)^{-3} \langle T \rangle$$

$$= -A^{-9}(-A^5 - A^{-3} + A^{-7})$$

$$\therefore \mathcal{L}_T = A^{-4} + A^{-12} - A^{-16}$$

$$\therefore \mathcal{L}_{T^*} = A^4 + A^{12} - A^{16}.$$

Expressing the Bracket as a Trace of a representation of the Artin Braid Group to the Temperley Lieb Algebra.

$$\begin{tikzcd}
B_n \arrow[r, "\mathrm{rep}"] \arrow[dr, "<\ \ >"'] & TL_n \arrow[d, "\mathrm{TR\ (loop\ count)}"] \\
 & Z[A,1/A]
\end{tikzcd}$$

$$<\mathrm{Closure}(b)> = \mathrm{TR}(\mathrm{rep}(b))$$

Since the 1980's it has been conjectured that $f(K)(A) = 1$ if and only if K is the unknot.

We conjecture that that the Jones polynomial detects the unknot.

In the late 1990's Mikhail Khovanov found an astonishing generalization of the bracket polynomial model of the Jones polynomial, replacing the polynomial by a homology theory whose graded Euler characteristic returned the polynomial.

In 2010 Kronheimer and Morwka proved that Khovanov Homology detects the unknot.

Their proof uses techniques from gauge theory and is related to mathematical physics.

We still do not know if the Jones polynomial detects the unknot.

Partition Functions in Statistical Mechanics

$$Z_G = \sum_{\sigma} e^{-E(\sigma)},$$

where σ runs over all "states" of the lattice G (we will let G be a planar graph) and $E(\sigma)$ is the energy of the given state. In the Potts model the energy has the form

$$E(\sigma) = \frac{1}{kT} \sum_{\langle i,\, j\rangle} \delta(\sigma_i, \sigma_j),$$

where $\langle i,\, j\rangle$ denotes an edge of G with vertices i, j and σ_i and σ_j are the state's assignments to these vertices. We assume that each vertex can be freely assigned one of q values, and that a state σ is such an assignment. In this formula δ is the Kronecker delta

$$\delta(a, b) = \begin{cases} 1 & \text{if } a = b \\ 0 & \text{otherwise} \end{cases}$$

and T is the *temperature* of the system, while k is a constant (Boltzman's constant).

Knots and the Potts Model

PROPOSITION 6.3. *For*

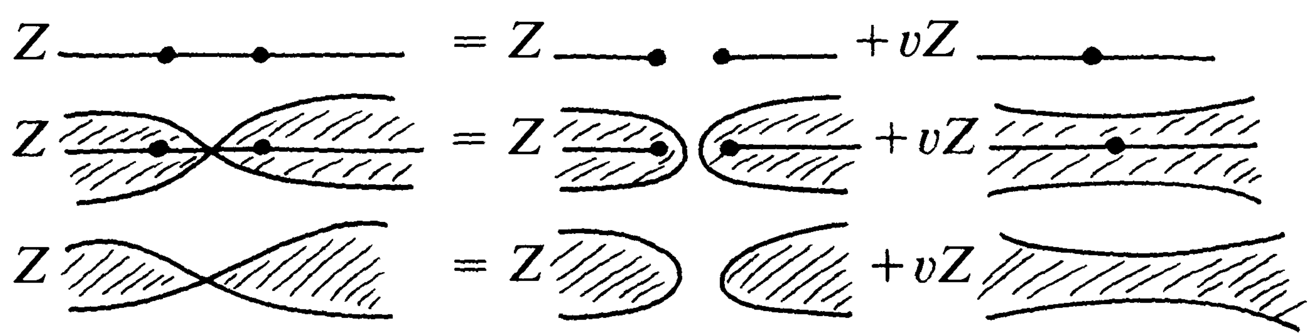
$$E(\sigma) = \frac{1}{kT} \sum_{\langle i, j \rangle} \delta(\sigma_i, \sigma_j)$$

and q local states, let

$$v = e^{-(1/kT)} - 1.$$

Then the partition function is the dichromatic polynomial in q and v:

$$\sum_{\sigma} e^{-E(\sigma)} = Z_G(q, v).$$

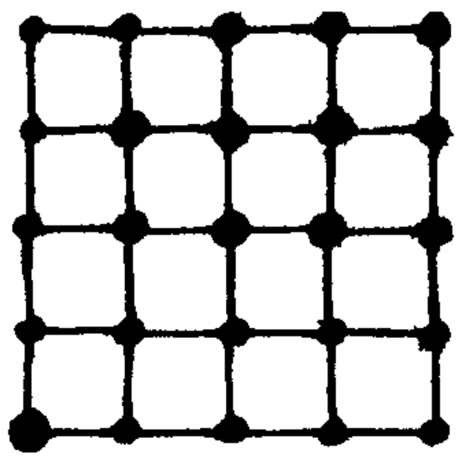


The dichromatic polynomial rewrites in bracket formalism, expressing the Potts model in terms of knot diagrams.

The Potts Model

The Potts Model.

In this model we are concerned with calculating a partition function associated with a planar graph G. In general for a graph G with vertices $i, j, \ldots$ and edges $\langle i, j \rangle$ a partition function has the form $Z_G = \sum_S e^{-E(S)/kT}$ where S runs over states of G, $E(S)$ is the energy of the state S, k is Boltzmann's constant and T is the temperature.



In the Potts model, the states involve choices of q values $(1, 2, 3, \ldots, q$ say) for each vertex of the graph G. Thus if G has N vertices, then there are q^N states. The q values could be spins of particles at sites in a lattice, types of metals in an alloy, and so on. The choice of spins (we'll call $1, \ldots, q$ the spins) at the vertices is free, and the energy of a state S with spin S_i at vertex i is taken to be given by

$$E(S) = \sum_{\langle i,j \rangle} \delta(S_i, S_j).$$

Here we sum over all edges in the graph. And $\delta(x, y)$ is the Kronecker delta:

$$\delta = (x, y) = \left\{ \begin{matrix} 1 & \text{if } x = y \\ 0 & \text{if } x \neq y \end{matrix} \right\}.$$

$$Z_G = \sum_S e^{K \sum_{\langle i,j \rangle} \delta(S_i, S_j)} \qquad (K = -1/kT)$$

$$= \sum_S \prod_{\langle i,j \rangle} e^{K\delta(S_i,S_j)}$$

$$\therefore \; Z_G = \sum_S \prod_{\langle i,j \rangle} (1 + v\delta(S_i, S_j))$$

where $v = e^K - 1$

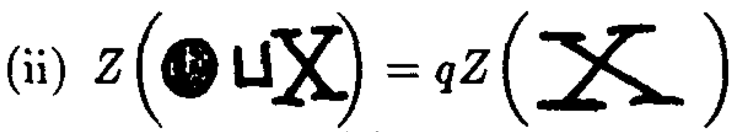
(i.e. $e^{K\delta(x,y)} = 1 + v\delta(x, y)$).

$$Z_G = \sum_S \prod_{\langle i,j \rangle} (1 + v\delta(S_i, S_j))$$

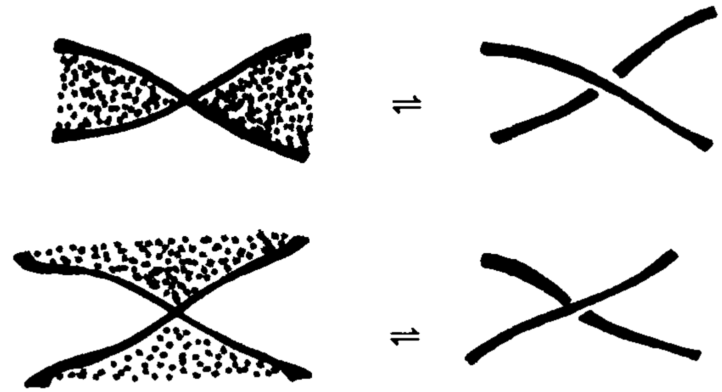
Dichromatic Graph Polynomial

$$Z(\bullet \sqcup H) = qZ(H).$$

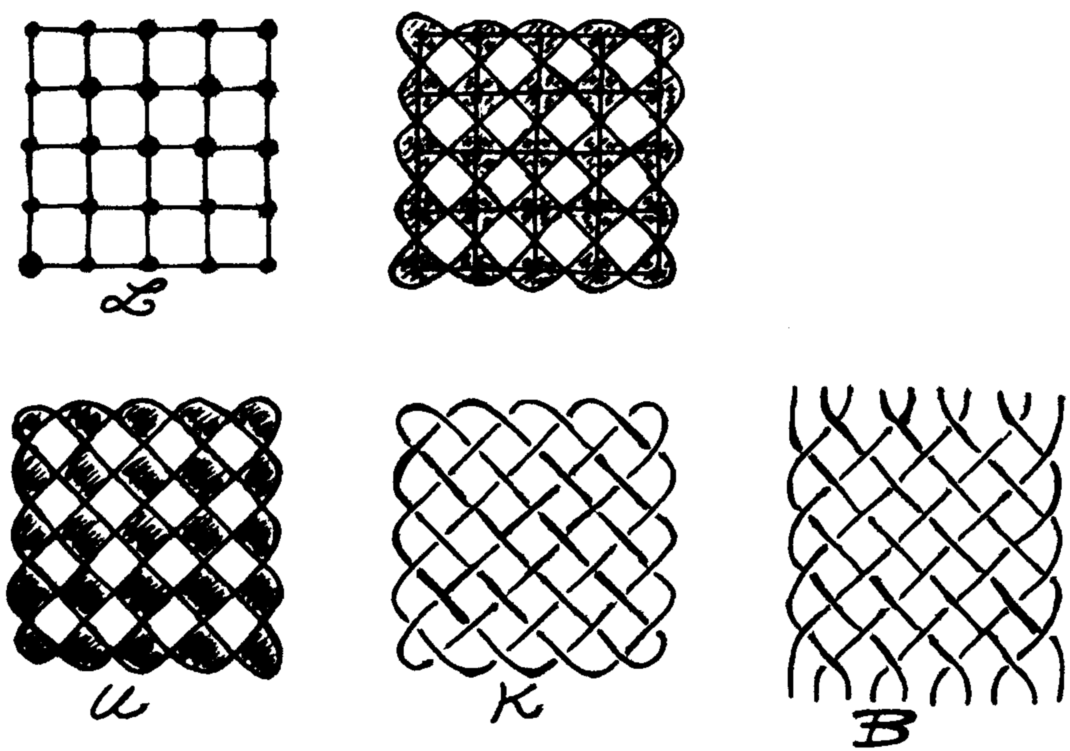
Translating to the Shaded Medial Graph:



. Here  stands for any connected shaded region, and $\bigsqcup$ denotes disjoint union.



Crossing Transcriptions of Local Shading



Since W satisfies

W() $= W$() $+ (q^{-1/2}v)W$()

W() $= q^{1/2}W(K).$

(Note the extra component is not shaded!) We can write the

W-Axioms (The Potts Bracket).

1. Let K be any knot or link diagram. Then $W(K) \in \mathbf{Z}[q^{1/2}, q^{-1/2}, v]$ is a well-defined function of q and v.

2. W() $= W$() $+ (q^{-1/2}v)W$()

   W() $= (q^{-1/2}v)W$() $+ W$()

3. W() $= q^{1/2}W(K)$

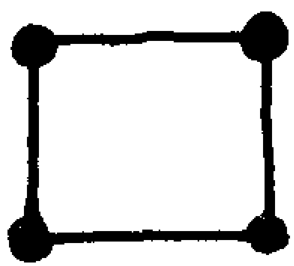
4. W() $= q^{1/2}.$

$$Z(K) = q^{N/2} W(K)$$

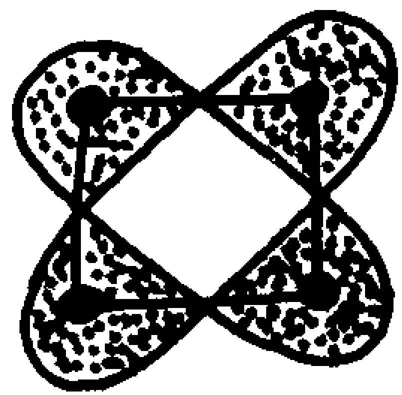
This translates the Potts model into a version of the bracket calculation on the associated alternating link diagram to a planar graph.

The relationship of the Potts Model with the Temperley Lieb algebra becomes transparent with this point of view.

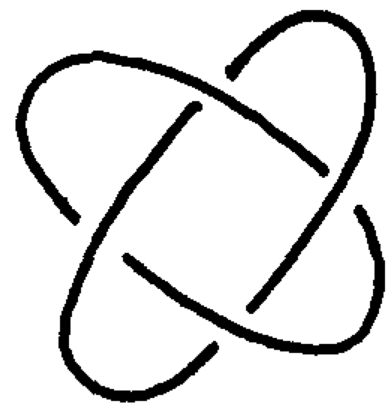
Exercise: Calculate the Potts model using the graph and using the link.



$\mathcal{L}$



U



$K = K(\mathcal{L})$

Critical Temperature

in the anti-ferromagnetic case of a large rectangular lattice one expects the critical point to occur when there is a symmetry between the partition function on the lattice and the dual lattice. In the bracket reformulation of the Potts model this corresponds to having $W($$) = W($$)$ and this occurs when $q^{-(1/2)}v = 1$, Hence the critical temperature occurs (conjecturally) at

$$e^{(1/kT)} - 1 = \sqrt{q}$$

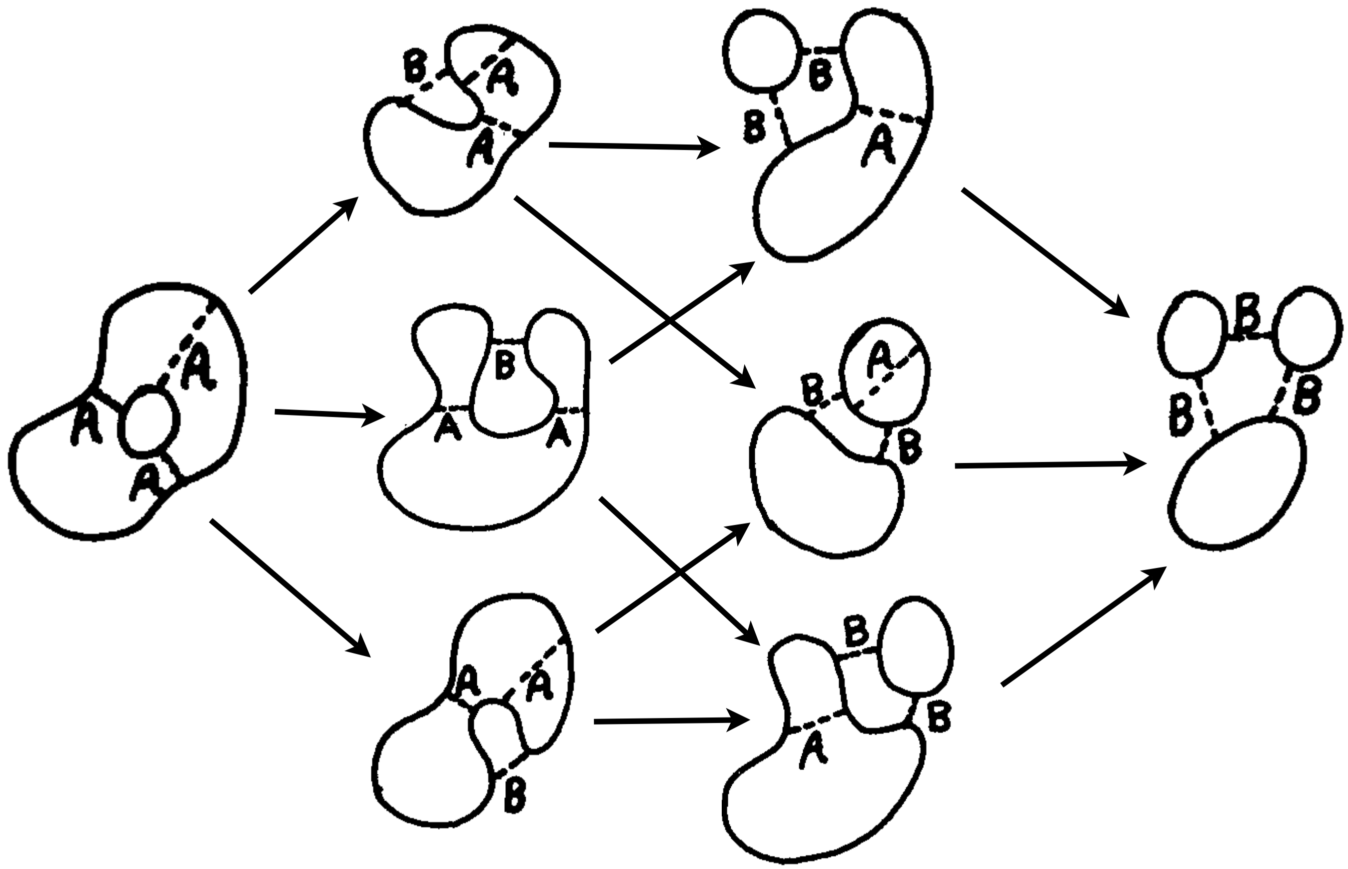
or

$$T = \frac{1}{k \ln(1 + \sqrt{q})} \; .$$

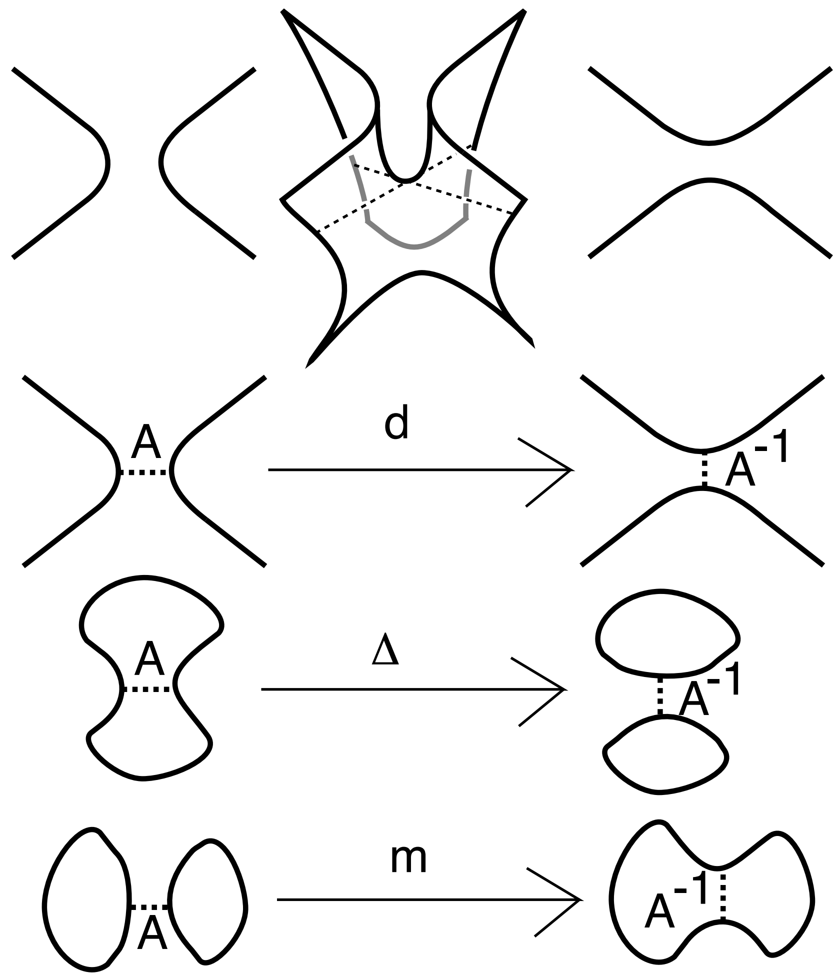
MANY QUESTIONS
About the relationship of
Knot Theory and the Potts Model
and
Statistical Mechanics

The Khovanov Complex

A Category of States



Khovanov Homology

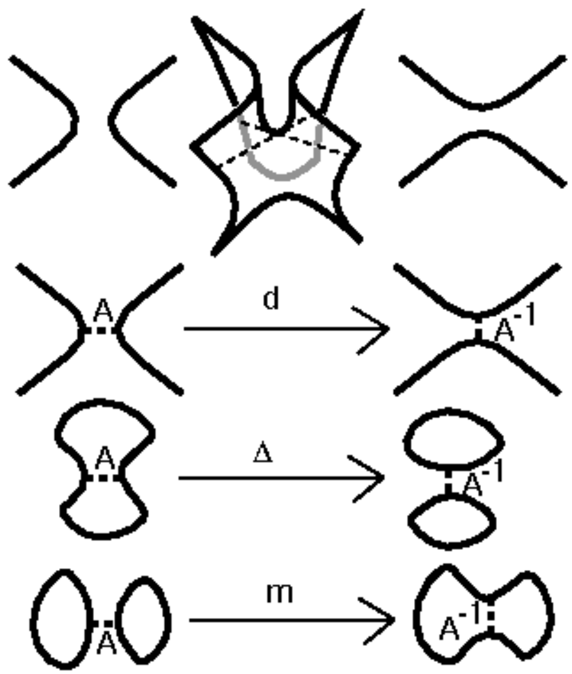


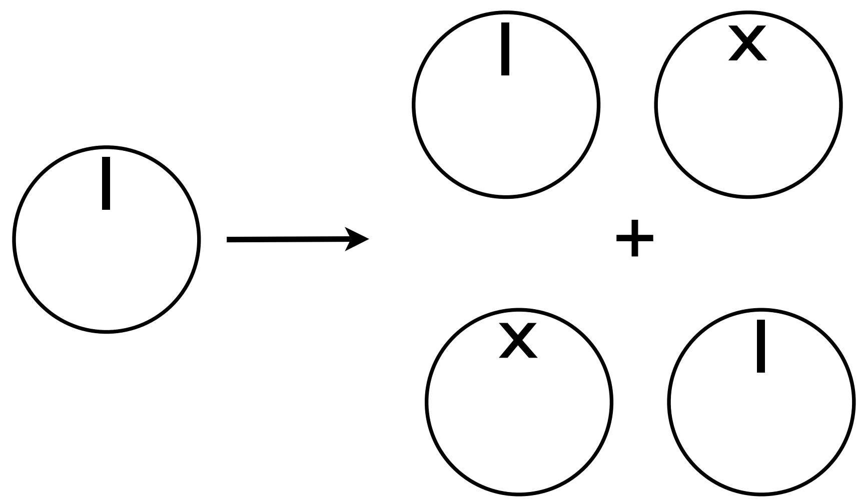
$$\partial(s) = \sum_{\tau} \partial_{\tau}(s)$$

The boundary is a sum of partial differentials corresponding to resmoothings on the states.

$\Delta(X) = X \otimes X$ and $\Delta(1) = 1 \otimes X + X \otimes 1.$

$X^2 = 0$





Cobordism Category

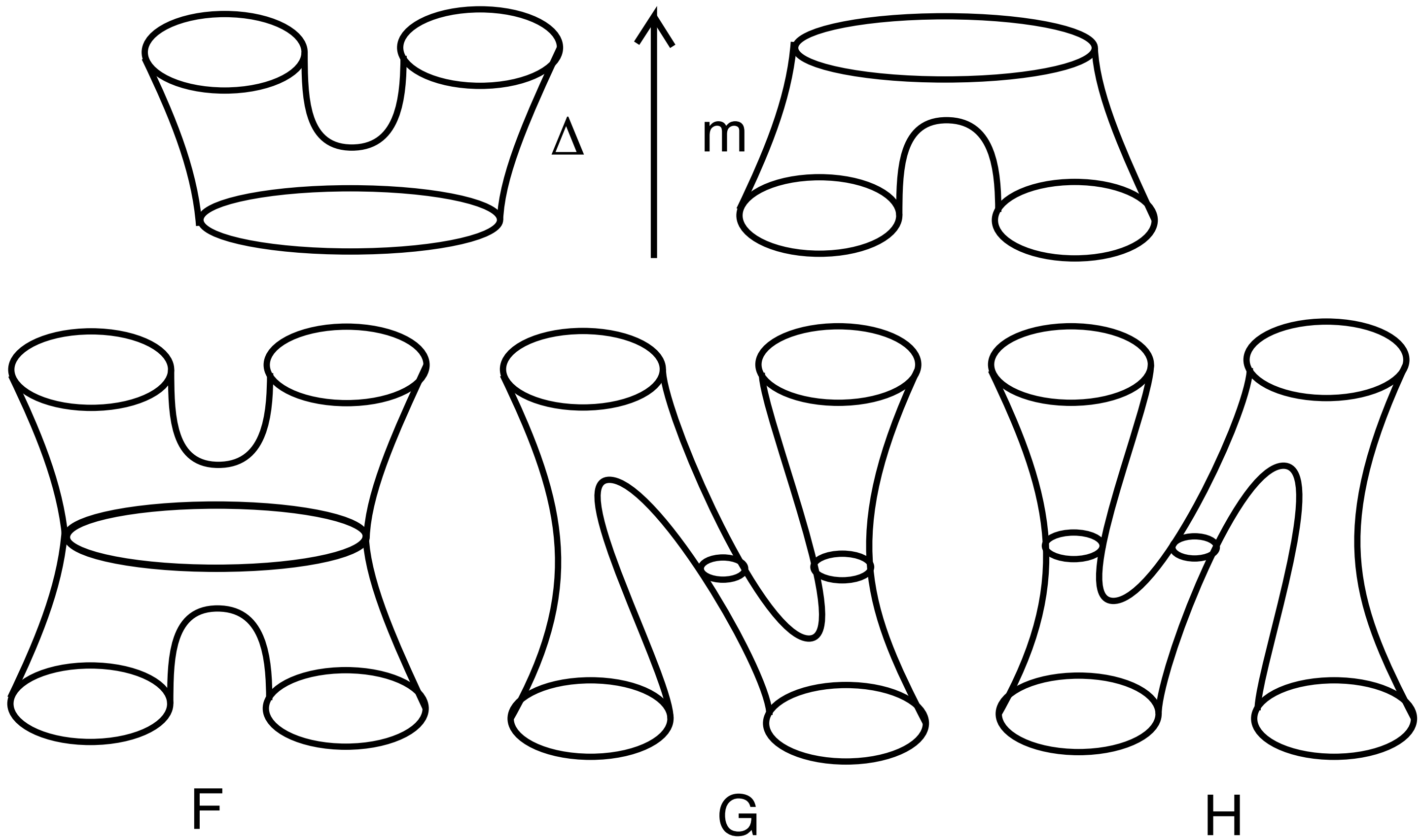
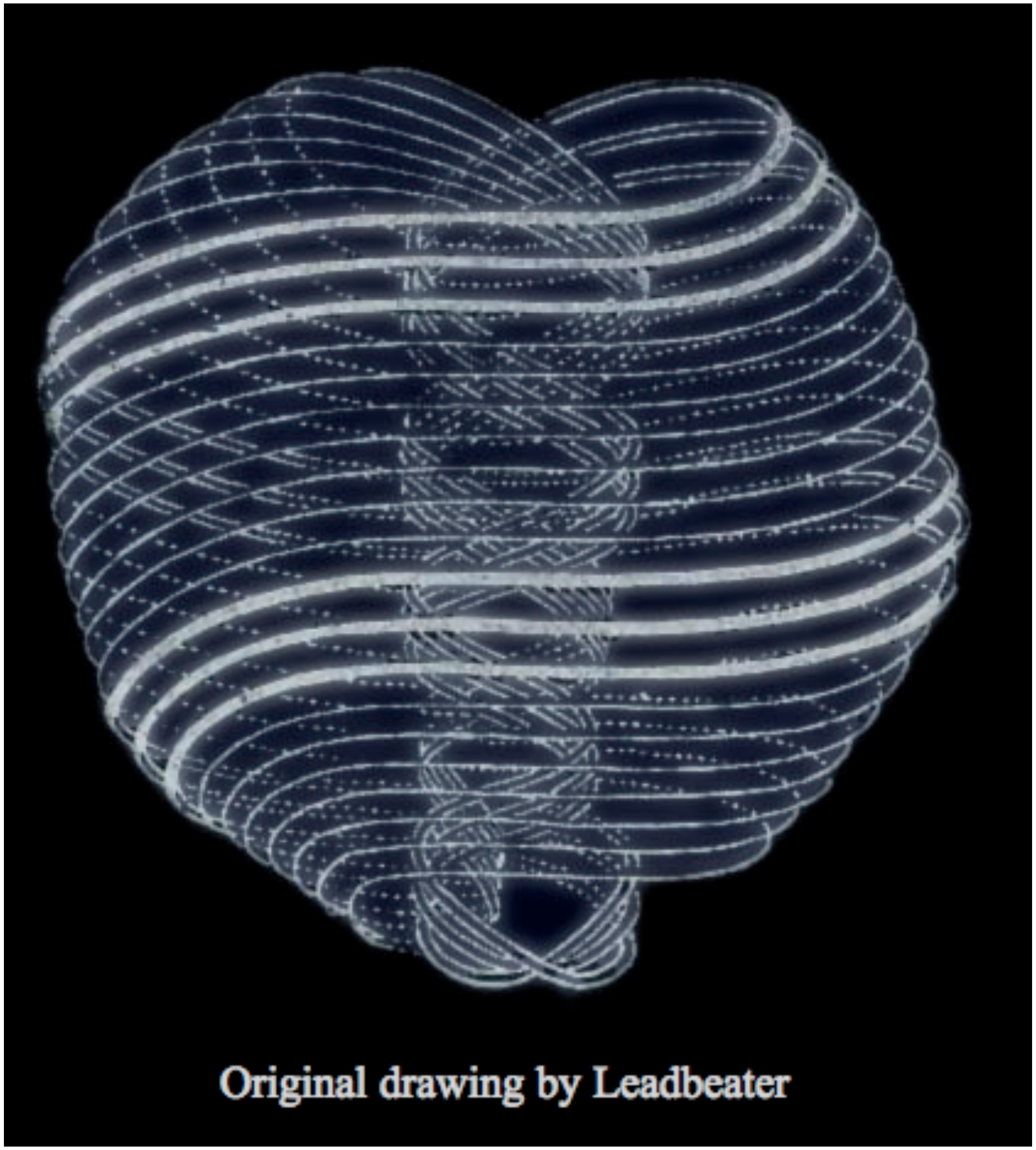


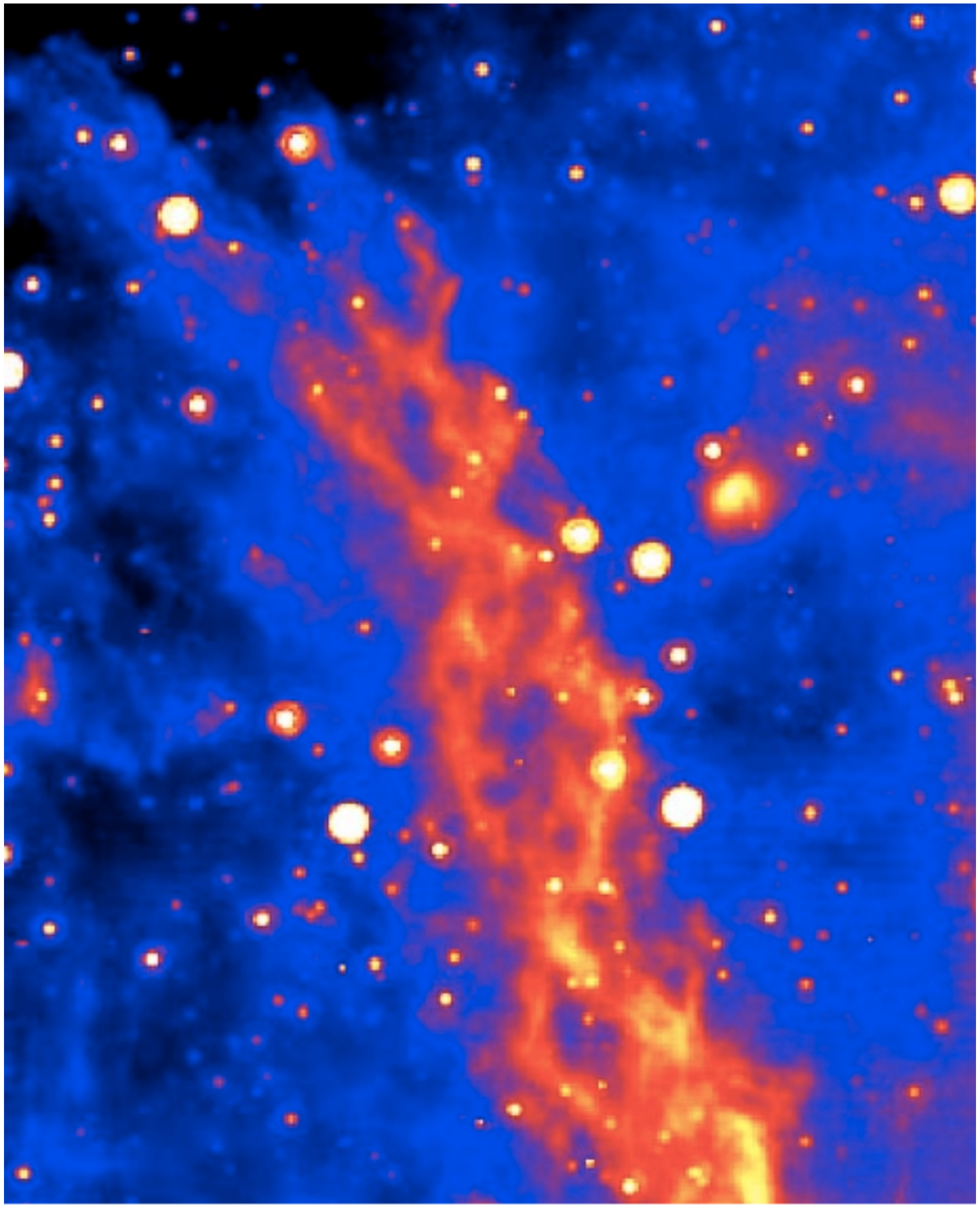
Figure 21 – The Frobenius Algebra Conditions

Categorification

Khovanov constructs a homology theory
that generalizes the bracket polynomial,
and such that the Jones polynomial
is the graded Euler characteristic
of this homology theory.



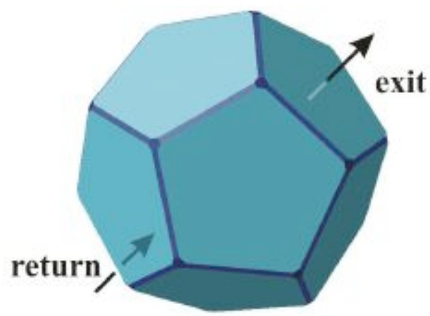
From the same period as Kelvin, the “vortex atom” of the visionaries Besant and Leadbeater.

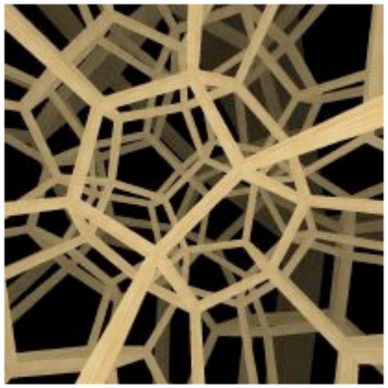


The DNA nebula
is an 80 light year
long formation
lying near the
enormous black hole
at the center of our
Milky Way galaxy.

http://news.nationalgeographic.com/news/2006/03/0317_060317_dna_nebula.html

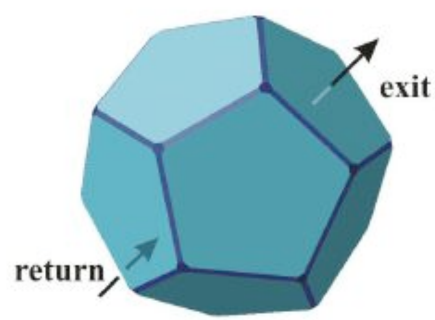
Is the Geometric Universe a Poincare Dodecahedral Space?!



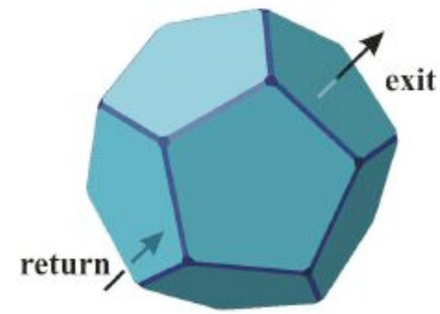


A franco-american team of cosmologists [1] led by J.-P. Luminet, of the Laboratoire Univers et Théories (LUTH) at the Paris Observatory, has proposed an explanation for a surprising detail observed in the Cosmic Microwave Background (CMB) recently mapped by the NASA satellite WMAP. According to the team, who published their study in the 9 October 2003 issue of Nature, an intriguing discrepancy in the temperature fluctuations in the afterglow of the big bang can be explained by a very specific global shape of space (a "topology"). The universe could be wrapped around, a little bit like a "soccer ball", the volume of which would represent only 80% of the observable universe! (figure 1) According to the leading cosmologist George Ellis, from Cape Town University (South Africa), who comments on this work in the "News & Views" section of the same issue: "If confirmed, it is a major discovery about the nature of the universe".

The Poincare Dodecahedral space is
obtained by identifying opposite
sides of a dodedahedron with
a twist.
The resulting space, if you were inside it,
would be something like the next slide.
Whenever you crossed a pentagonal face,
you would find yourself back in
the Dodecahedron.



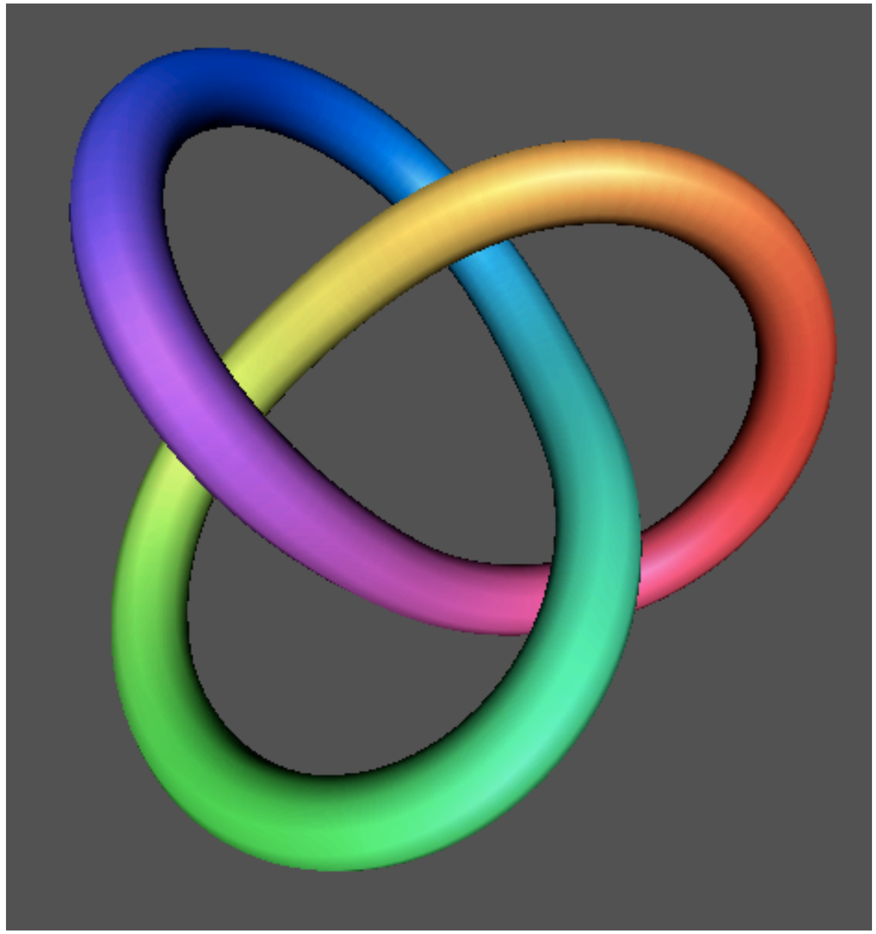
What Does This Have
to do with Knot Theory?



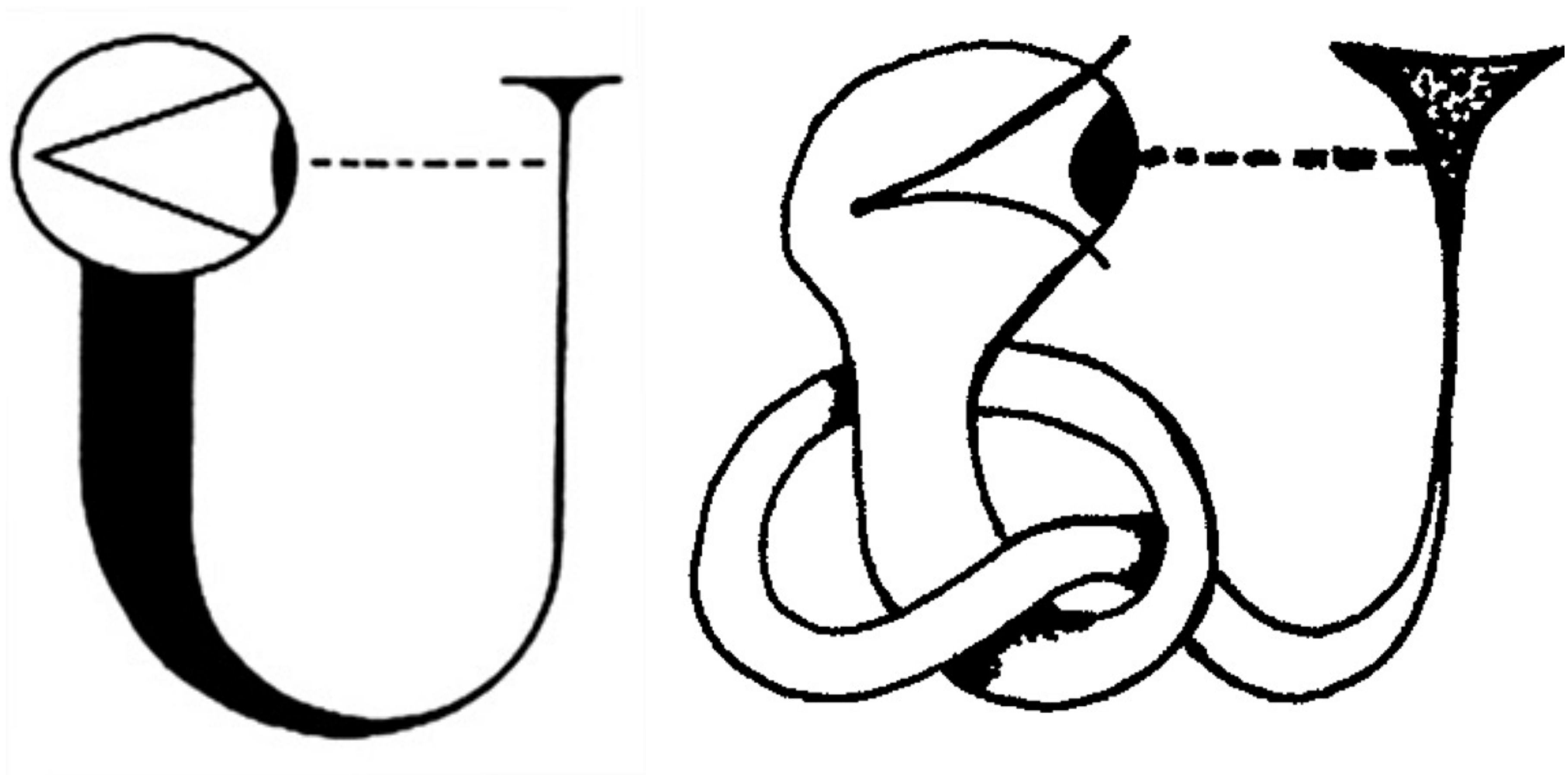
The Dodecahedral Space has
Axes of Symmetry:
five-fold, three-fold and two-fold.

If you identify points that are symmetrically
placed with respect to the five-fold
symmetry axis, the space folds up to become
a three-dimensional sphere, and
the axis becomes a trefoil knot in that sphere!

We say that the Dodecahedral space is the
5-fold cyclic branched covering
of the Three-Sphere, branched along the
trefoil knot.



So perhaps the trefoil knot is the
key to the universe.



Is the Universe Knotted?

Are elementary particles knotted quantized flux?

PHYSICAL REVIEW D VOLUME 6, NUMBER 2 15 JULY 1972

Flux Quantization and Particle Physics

Herbert Jehle

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(Received 27 September 1971; revised manuscript received 27 December 1971)

Quantized flux has provided an interesting model for muons and for electrons: One closed flux loop of the form of a magnetic dipole field line is assumed to adopt alternative forms which are superposed with complex probability amplitudes to define the magnetic field of a source lepton. The spinning of that loop with an angular velocity equal to the *Zitterbewegung* frequency $2mc^2/\hbar$ implies an electric Coulomb field, (negative) positive, depending on (anti) parallelism of magnetic moment and spin. The model implies CP invariance. A quark may be represented by a quantized flux loop if interlinked with another loop in the case of a meson, with two other loops in the case of a baryon. Because of the link, their spinning is very different from that of a single loop (lepton). The concept of a single quark does not exist accordingly, and it is seen that a baryon with a symmetric spin-isospin function in the SU(2) $\times$ SU(3) quark representation might not violate the Pauli principle because the wave function representing the relative position of linked loops may be chosen antisymmetric. Weak interactions may be understood to occur when the flux loops involved in the interaction have to cross over themselves or over each other. Strangeness is readily interpreted in terms of the trefoil character of a λ quark: Strangeness-violating interactions imply crossing of flux lines and are thus weak and parity-nonconserving. $\Delta S = \Delta Q$ is favored in such interactions. Intrinsic symmetries may be interpreted in terms of topology of linked loops. Sections I and II give a short résumé of the 1971 paper.

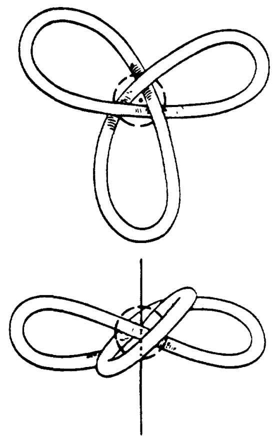

FIG. 2. A trefoil representing a neutrino loop which, like a coasting three-bladed propeller, moves in a helical spinning motion in the direction of the spin axis. In this and in subsequent figures, flux loops are drawn as double lines merely to better visualize the form of the loops. The loops are singular lines, the alternative forms of which define fibration of space. The question of orientation of the magnetic flux is still open; a neutrino might even be a superposition, not only of different loopforms, but also of both signatures of magnetic flux orientation. The difference between electron and muon neutrino is discussed in Sec. IV and in Appendix II of Ref. 1; the distinction is in regard to phase-related versus random-phased probability amplitudes superposition of the contributions of loopform bundles. A *single* loop of this form never represents anything else but a neutrino.

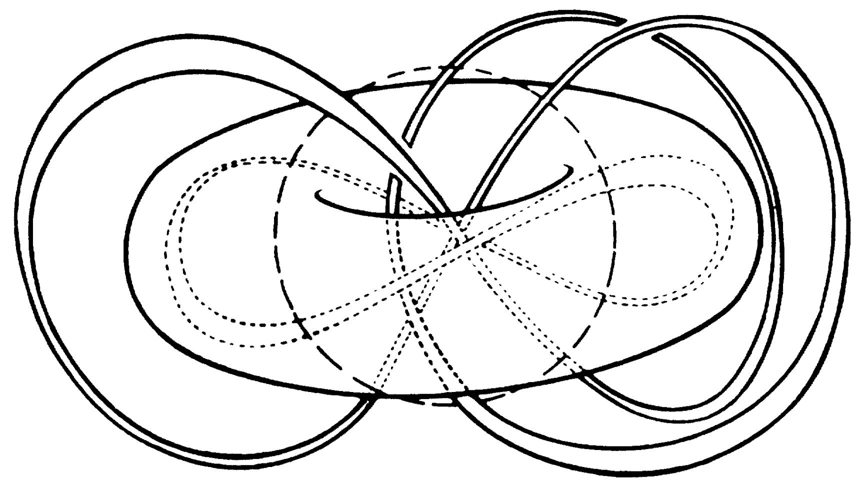

FIG. 4. Spinning-top model. λ and $\overline{\mathfrak{N}}$ quark interlinked, contributing to a meson. To illustrate the topological (knot-theoretical) relationships of the two loops, space is here subdivided by a toroidal surface [dashed lines in Fig. 4(a) which show a doughnut cut in half]. The λ is located entirely outside this doughnut shaped surface, the $\overline{\mathfrak{N}}$ entirely inside. This surface is dividing the fibrated space of λ loopforms from that of $\overline{\mathfrak{N}}$ loopforms; this toroidal interface may arbitrarily shrink or extend itself. Both loops pass through the spherical core region which is indicated by the dashed circle; the two loops may spin independently in a rolling-spinning motion about both the circular and the straight axes.

Jumping forward many years:

Protons are made of quarks.
Quarks are bound by gluon field.
Glueballs are closed loops of gluon field.

Can glueballs be knotted?!

Are Glueballs Knotted Closed Strings?

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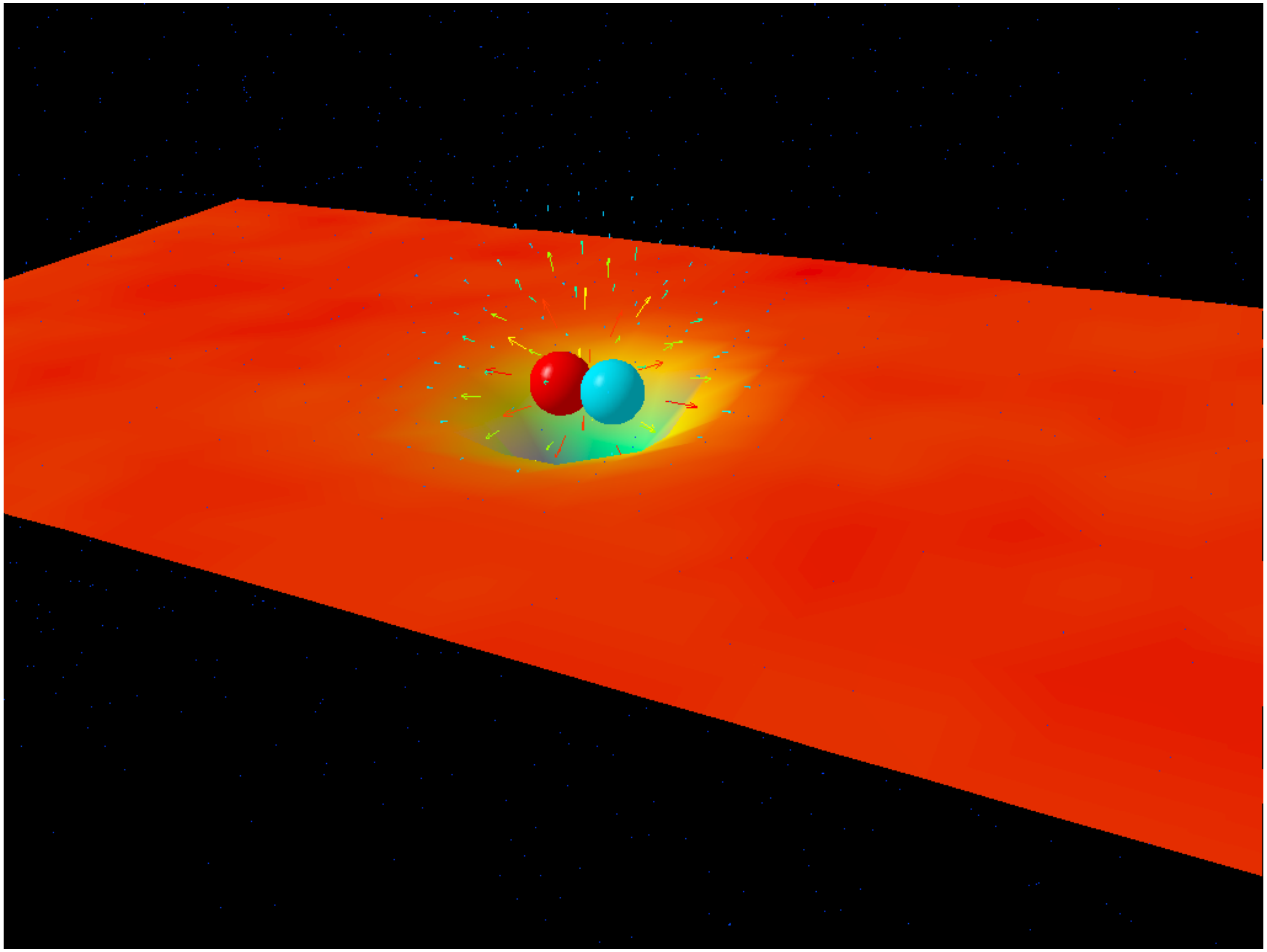
May 29, 2006

Abstract

Glueballs have a natural interpretation as closed strings in Yang-Mills theory. Their stability requires that the string carries a nontrivial twist, or then it is knotted. Since a twist can be either left-handed or right-handed, this implies that the glueball spectrum must be degenerate. This degeneracy becomes consistent with experimental observations, when we identify the $\eta_L(1410)$ component of the $\eta(1440)$ pseudoscalar as a 0^{-+} glueball, degenerate in mass with the widely accepted 0^{++} glueball $f_0(1500)$. In addition of qualitative similarities, we find that these two states also share quantitative similarity in terms of equal production ratios, which we view as further evidence that their structures must be very similar. We explain how our string picture of glueballs can be obtained from Yang-Mills theory, by employing a decomposed gauge field. We also consider various experimental consequences of our proposal, including the interactions between glueballs and quarks and the possibility to employ glueballs as probes for extra dimensions: The coupling of strong interactions to higher dimensions seems to imply that absolute color confinement becomes lost.



Universal energy spectrum of tight knots and links in physics*

Roman V. Buniy† and Thomas W. Kephart‡
Department of Physics and Astronomy, Vanderbilt University, Nashville, TN 37235, USA

We argue that a systems of tightly knotted, linked, or braided flux tubes will have a universal mass-energy spectrum, since the length of fixed radius flux tubes depend only on the topology of the configuration. We motivate the discussion with plasma physics examples, then concentrate on the model of glueballs as knotted QCD flux tubes. Other applications will also be discussed.

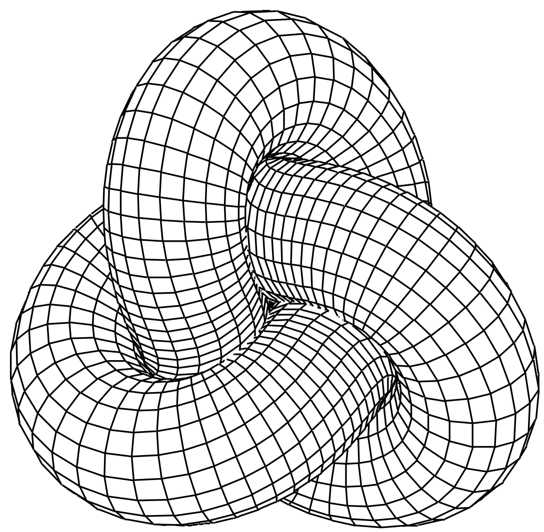
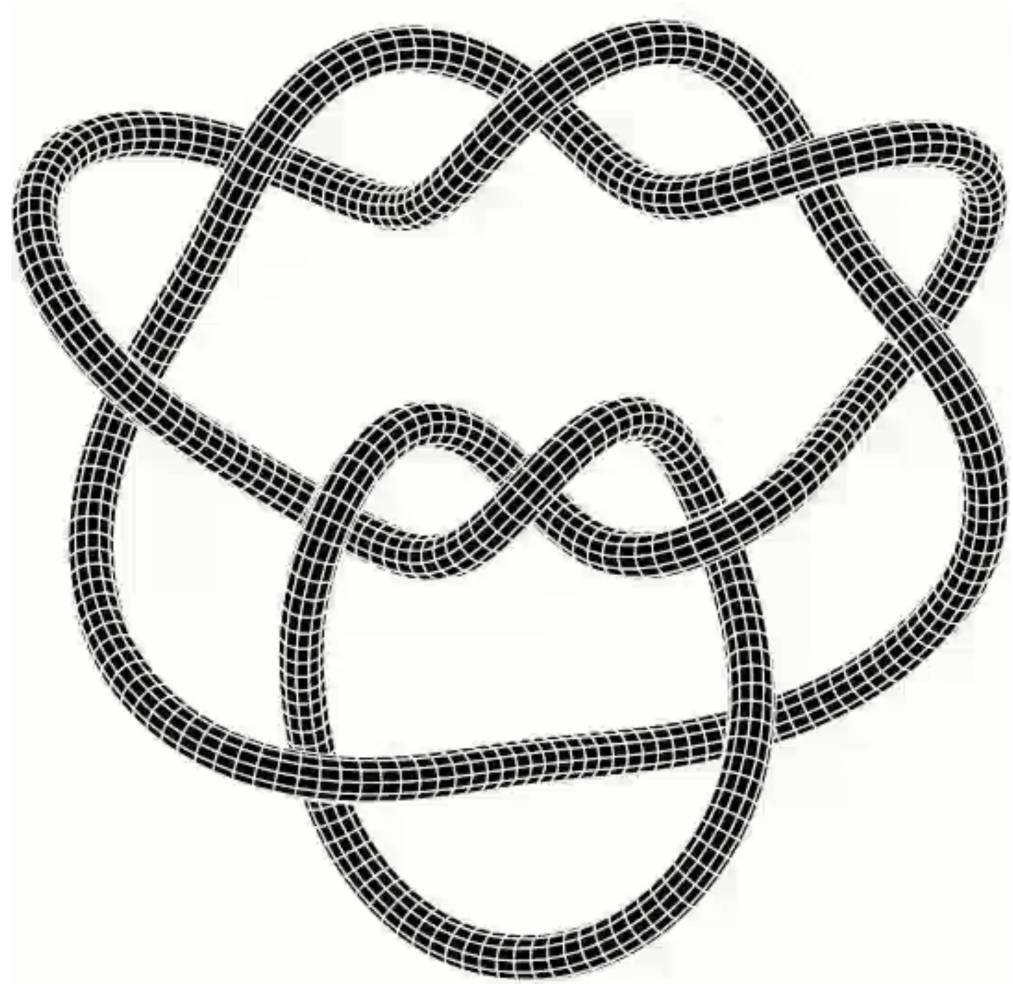


Figure 2: The second shortest solitonic flux configuration is the trefoil knot 3_1 corresponding to the second lightest glueball candidate $f_0(980)$.

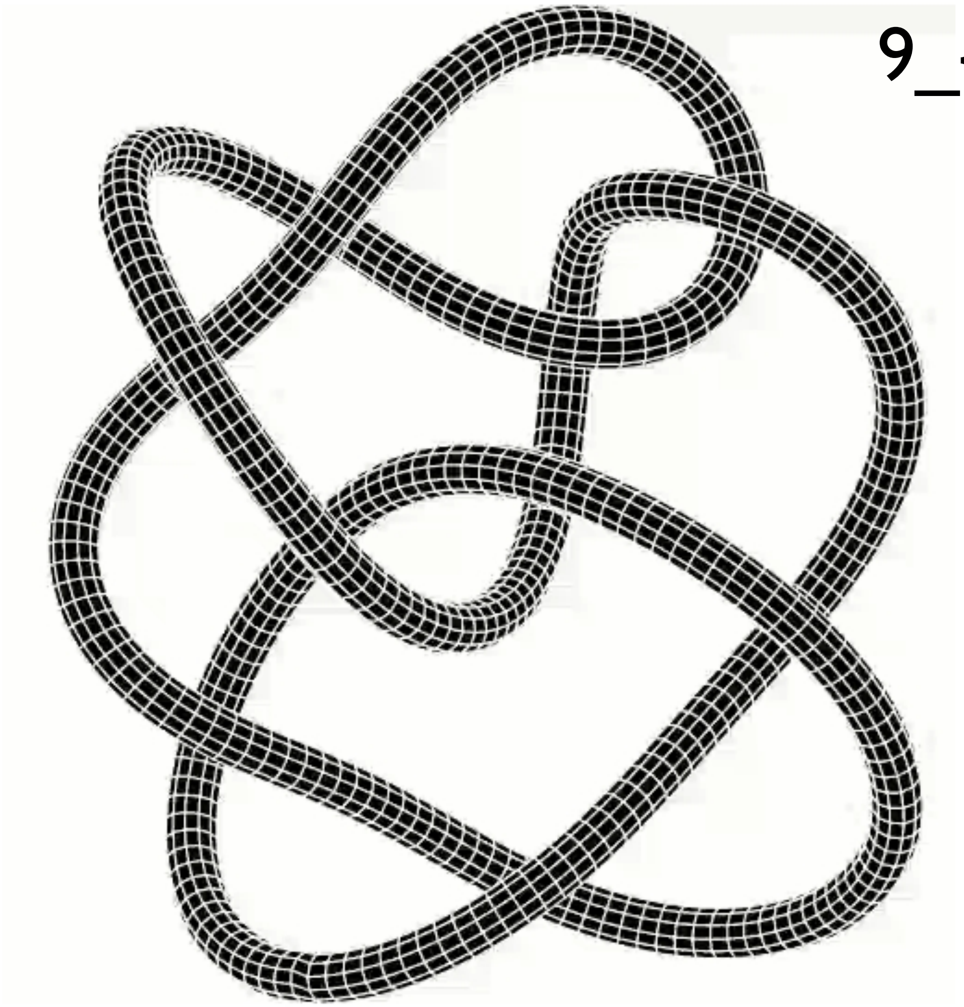


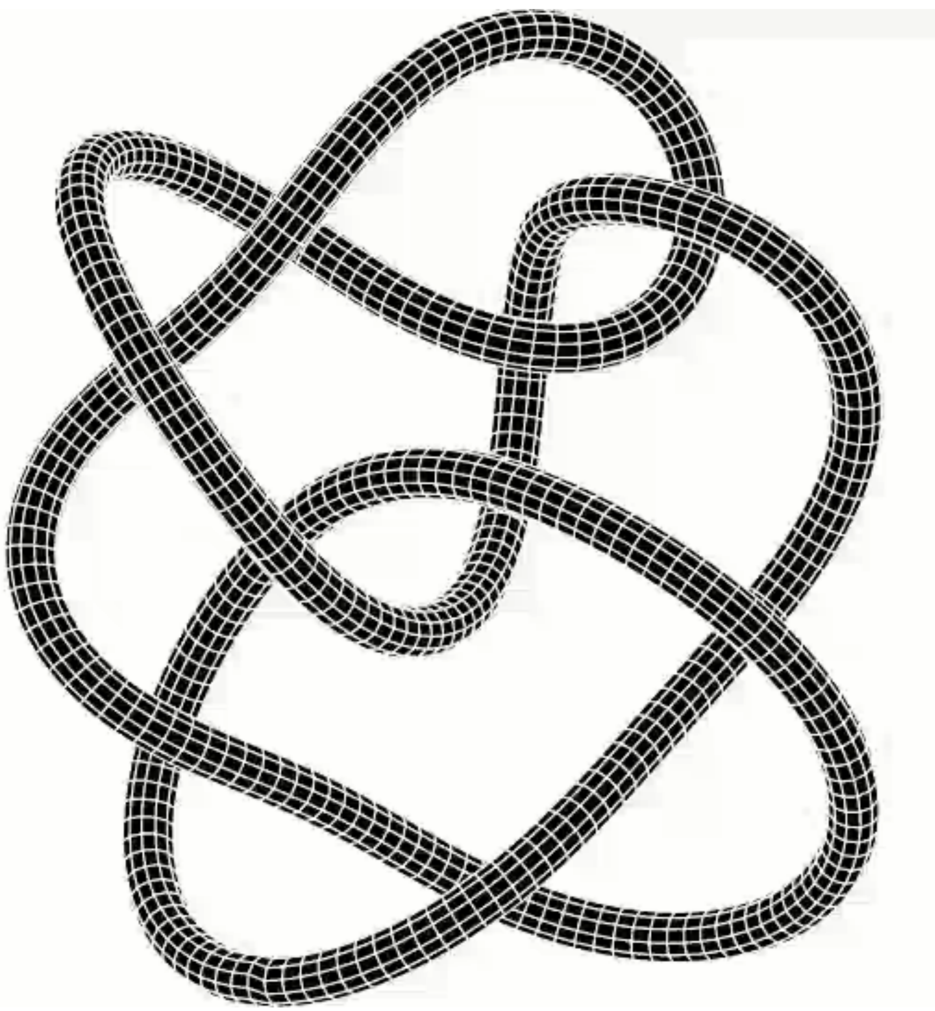
The previous demonstration as made by Jason Cantarella, using his program “ridgerunner”.

http://www.math.uga.edu/~cantarel/

In the next frame we show another Cantarella film, contracting the knot 9_{42}. This is the first chiral knot that is undetected from its mirror image by the Jones polynomial.

9_{42}





A topological model of composite preons

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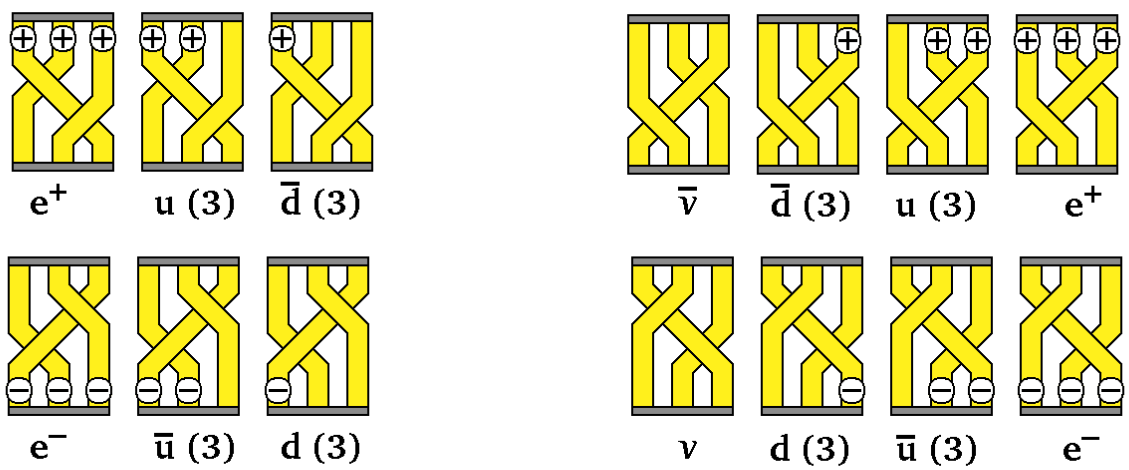
Centre for the Subatomic Structure of Matter, Department of Physics,
University of Adelaide, Adelaide SA 5005, Australia
(Dated: October 27, 2006)

We describe a simple model, based on the preon model of Shupe and Harari, in which the binding of preons is represented topologically. We then demonstrate a direct correspondence between this model and much of the known phenomenology of the Standard Model. In particular we identify the substructure of quarks, leptons and gauge bosons with elements of the braid group B_3. Importantly, the preonic objects of this model require fewer assumed properties than in the Shupe/Harari model, yet more emergent quantities, such as helicity, hypercharge, and so on, are found. Simple topological processes are identified with electroweak interactions and conservation laws. The objects which play the role of preons in this model may occur as topological structures in a more comprehensive theory, and may themselves be viewed as composite, being formed of truly fundamental sub-components, representing exactly two levels of substructure within quarks and leptons.



Positron

Electron

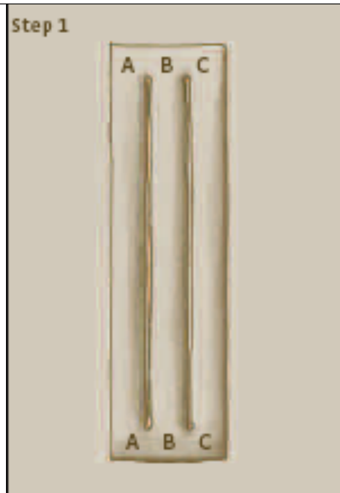
Down quark

Up quark

The Braided Belt Trick

The mathematics of Sundance Bilson's approach to elementary particles based on the 'braided belt trick" shown in the next slide.

This trick is also the basis for making braided leather belts.

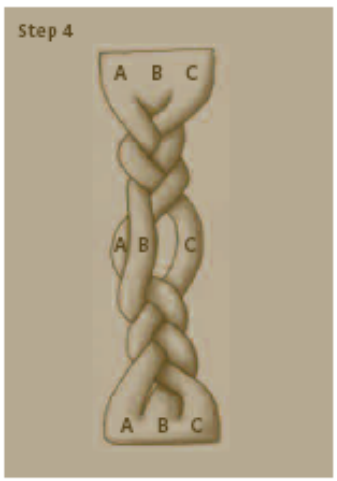


Begin by cutting two slits into a strip of leather.

Be careful not to cut all the way to the ends.

Holding the top flat, pull string C over string B, and pull string A over string C.

Next, pull string B over string A, and pull string C over string B.

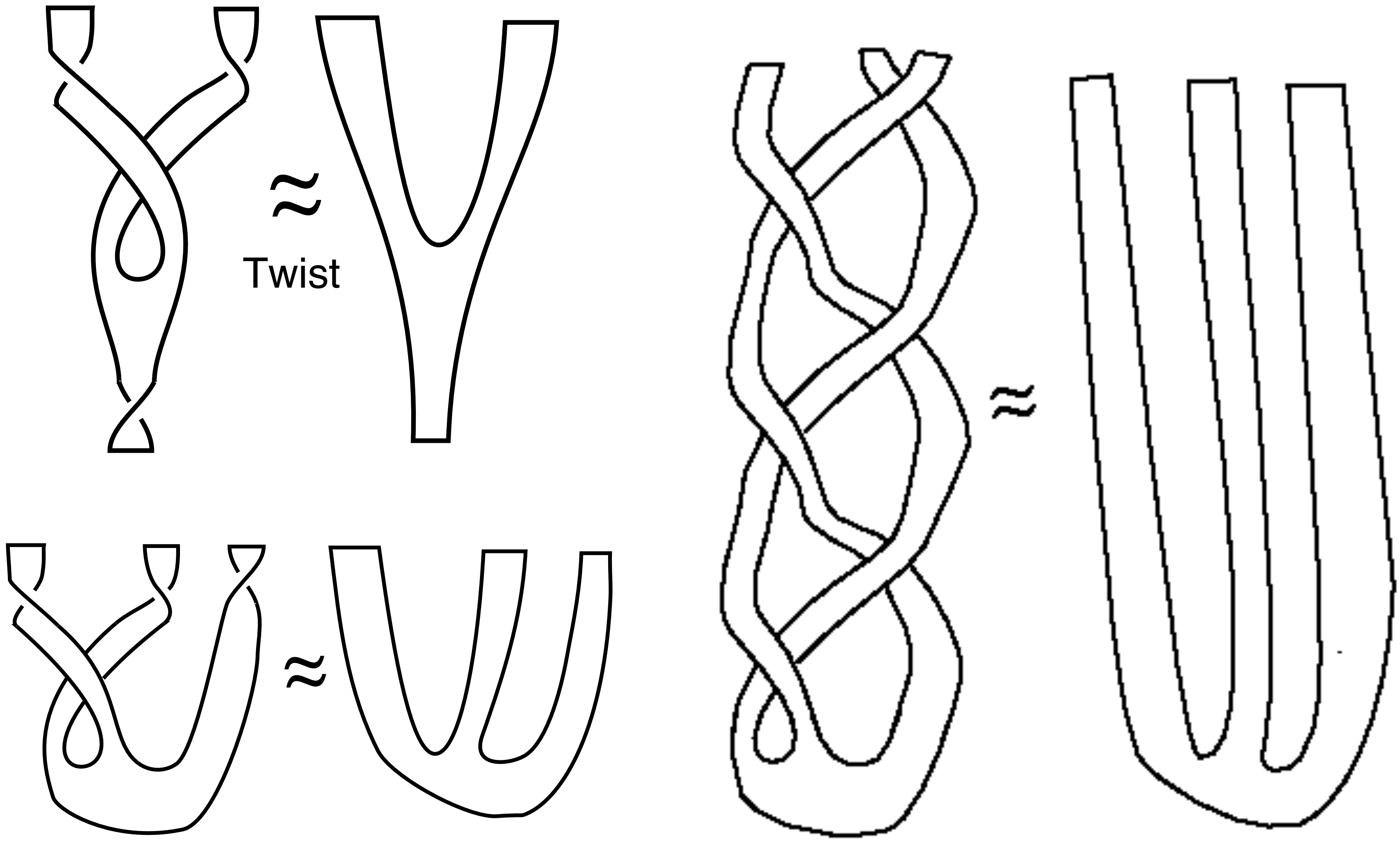


Now pull string A over string C, and pull string B over string A.

Untangle the bottom portion by sliding the bottom end through the open slits.

Continue this pattern until the braid reaches the bottom of the strip.

The Braided Belt Trick



This approach to elementary particle
physics is just beginning.
We will have to wait and see
if elementary particles are
braids and if knotted glueballs
are real.

After all,
Why Knot?

Knots and Quantum Field Theory

From Feynman's Nobel Lecture

The character of quantum mechanics of the day was to write things in the famous Hamiltonian way - in the form of a differential equation, which described how the wave function changes from instant to instant, and in terms of an operator, H. If the classical physics could be reduced to a Hamiltonian form, everything was all right. Now, least action does not imply a Hamiltonian form if the action is a function of anything more than positions and velocities at the same moment. If the action is of the form of the integral of a function, (usually called the Lagrangian) of the velocities and positions at the same time

$$S = \int L(\dot{x}, x)\, dt$$

then you can start with the Lagrangian and then create a Hamiltonian and work out the quantum mechanics, more or less uniquely. But this thing (1) involves the key variables, positions, at two different times and therefore, it was not obvious what to do to make the quantum-mechanical analogue.

L = Kinetic Energy - Potential Energy

Classical Mechanics: Extremize Integral of L over the paths from A to B.

So that didn't help me very much, but when I was struggling with this problem, I went to a beer party in the Nassau Tavern in Princeton. There was a gentleman, newly arrived from Europe (Herbert Jehle) who came and sat next to me. Europeans are much more serious than we are in America because they think that a good place to discuss intellectual matters is a beer party. So, he sat by me and asked, "what are you doing" and so on, and I said, "I'm drinking beer." Then I realized that he wanted to know what work I was doing and I told him I was struggling with this problem, and I simply turned to him and said, "listen, do you know any way of doing quantum mechanics, starting with action - where the action integral comes into the quantum mechanics?" "No", he said, "but Dirac has a paper in which the Lagrangian, at least, comes into quantum mechanics. I will show it to you tomorrow."

Next day we went to the Princeton Library, they have little rooms on the side to discuss things, and he showed me this paper. What Dirac said was the following: There is in quantum mechanics a very important quantity which carries the wave function from one time to another, besides the differential equation but equivalent to it, a kind of a kernal, which we might call $K(x', x)$, which carries the wave function j(x) known at time t, to the wave function j(x') at time, t+e Dirac points out that this function K was *analogous* to the quantity in classical mechanics that you would calculate if you took the exponential of ie, multiplied by the Lagrangian $L(\dot{x},x)$ imagining that these two positions x,x' corresponded t and t+e. In other words,

$$K(x',x) \text{ is analogous to } e^{i\varepsilon L\left(\frac{x'-x}{\varepsilon},x\right)/\hbar}$$

Professor Jehle showed me this, I read it, he explained it to me, and I said, "what does he mean, they are analogous; what does that mean, *analogous*? What is the use of that?" He said, "you Americans! You always want to find a use for everything!" I said, that I thought that Dirac must mean that they were equal. "No", he explained, "he doesn't mean they are equal." "Well", I said, "let's see what happens if we make them equal."

So I simply put them equal, taking the simplest example where the Lagrangian is ½$M\dot{x}^2$ - $V(x)$ but soon found I had to put a constant of proportionality A in, suitably adjusted. When I substituted $Ae^{i\epsilon L/\hbar}$ for K to get

$$\psi(x', t+\varepsilon) = \int A \exp\left[\frac{i\varepsilon}{\hbar} L\left(\frac{x'-x}{\varepsilon}, x\right)\right] \psi(x, t)\, \mathrm{d}x$$

and just calculated things out by Taylor series expansion, out came the Schrödinger equation. So, I turned to Professor Jehle, not really understanding, and said, "well, you see Professor Dirac meant that they were proportional." Professor Jehle's eyes were bugging out - he had taken out a little notebook and was rapidly copying it down from the blackboard, and said, "no, no, this is an important discovery. You Americans are always trying to find out how something can be used. That's a good way to discover things!" So, I thought I was finding out what Dirac meant, but, as a matter of fact, had made the discovery that what Dirac thought was analogous, was, in fact, equal. I had then, at least, the connection between the Lagrangian and quantum mechanics, but still with wave functions and infinitesimal times.

The Taylor expansion is

$$\psi(x,t+\epsilon) = \frac{e^{\frac{-i\epsilon V(x)}{\hbar}}}{A} \int e^{\frac{im\eta^2}{\hbar 2\epsilon}} [\psi(x,t) + \eta \frac{\partial \psi(x,t)}{\partial x} + \frac{\eta^2}{2} \frac{\partial^2 \psi(x,t)}{\partial x^2} + \cdots] d\eta.$$

Now use the Gaussian integrals

$$\int_{-\infty}^{\infty} e^{\frac{im\eta^2}{\hbar 2\epsilon}} d\eta = \sqrt{\frac{2\pi\hbar\epsilon i}{m}},$$

and

$$\int_{-\infty}^{\infty} \eta^2 e^{\frac{im\eta^2}{\hbar 2\epsilon}} d\eta = \sqrt{\frac{2\pi\hbar\epsilon i}{m}} \frac{\hbar\epsilon i}{m}.$$

This rewrites the Taylor series as follows.

$$\psi(x,t+\epsilon) = \frac{\sqrt{\frac{2\pi\hbar\epsilon i}{m}}}{A} e^{\frac{-i\epsilon V(x)}{\hbar}} [\psi(x,t) + \frac{\hbar\epsilon i}{2m} \frac{\partial^2 \psi}{\partial x^2} + O(x^2)].$$

Taking

$$A(\epsilon) = \sqrt{\frac{2\pi\hbar\epsilon i}{m}},$$

we get

$$\psi(x,t) + \epsilon \partial \psi(x,t)/\partial t = \psi(x,t) - \frac{i\epsilon}{\hbar} V(x)\psi(x,t) + \frac{\hbar i \epsilon}{2m} \partial^2 \psi / \partial x^2.$$

Hence $\psi(x,t)$ satisfies the Schrödinger equation.

Witten's Integral

In [49] Edward Witten proposed a formulation of a class of 3-manifold invariants as generalized Feynman integrals taking the form $Z(M)$ where

$$Z(M) = \int DAe^{(ik/4\pi)S(M,A)}.$$

Here M denotes a 3-manifold without boundary and A is a gauge field (also called a gauge potential or gauge connection) defined on M. The gauge field is a one-form on a trivial G-bundle over M with values in a representation of the Lie algebra of G. The group G corresponding to this Lie algebra is said to be the gauge group. In this integral the action $S(M, A)$ is taken to be the integral over M of the trace of the Chern-Simons three-form $A \wedge dA + (2/3)A \wedge A \wedge A$. (The product is the wedge product of differential forms.)

With the help of the Wilson loop functional on knots and links, Witten writes down a functional integral for link invariants in a 3-manifold M:

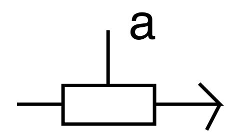
$$Z(M,K) = \int DAe^{(ik/4\pi)S(M,A)} tr(Pe^{\oint_K A})$$

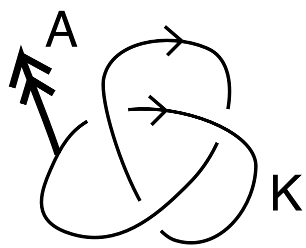
$$= \int DAe^{(ik/4\pi)S} < K|A > .$$

$$A(x) = A_a^k(x)T^a dx_k$$

The gauge field is a Lie-algebra valued one-form on 3-space.

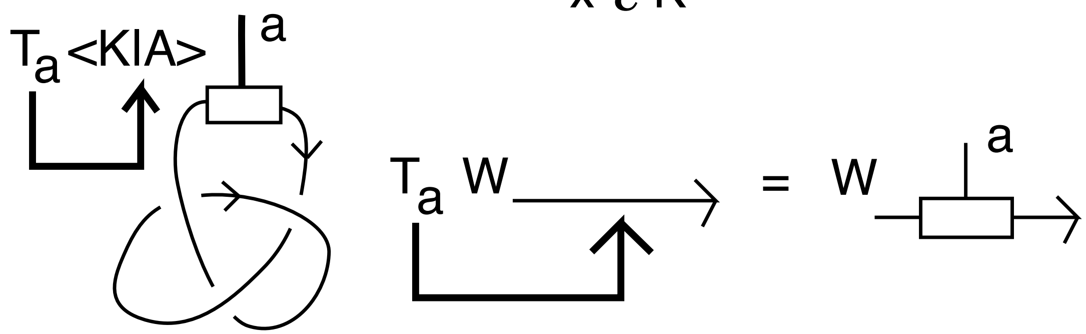
The next slide discusses the nature of the Wilson Loop.

$V \xrightarrow{T^a} V \Longleftrightarrow$ 



$$W_K(A) = \langle K|A\rangle = \mathrm{tr}(Pe^{\oint_K A})$$

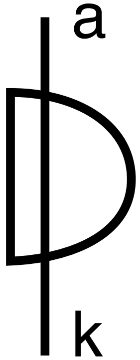
$$= \prod_{x \,\varepsilon\, K} (1 + A_a^i(x) T^a dx_i)$$



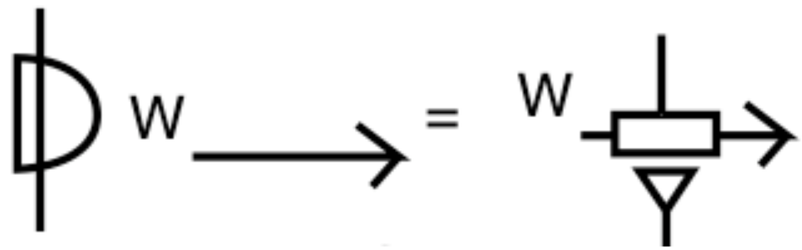
Think of a vector on the knot. As the base of the vector moves by dx the vector changes to (I + A)v. This is the analog of parallel translation. The gauge field is a connection!

$= dx_k$

$$W_K(A) = \langle K|A\rangle = tr(Pe^{\oint_K A}) = \prod_{x \varepsilon K} (1 + A_a^i(x) T^a dx_i)$$

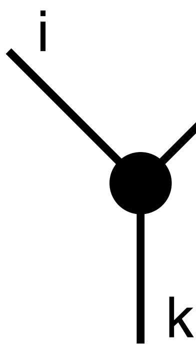


$= \delta/\delta A_a^k(x)$

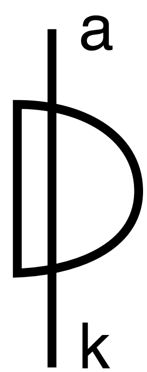


This diagram defines a symbol for dx_k.

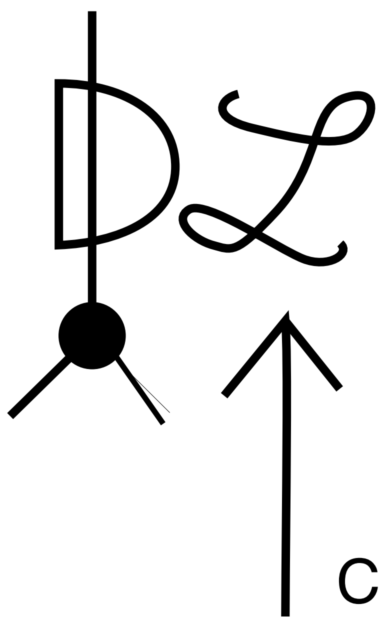
It shows the formula for differentiating a Wilson loop.



$= \varepsilon_{ijk}$



$= \delta/\delta A_a^k(x)$



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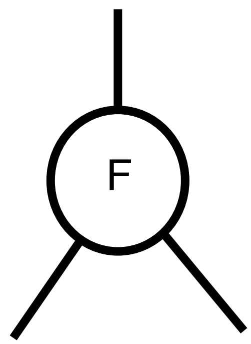


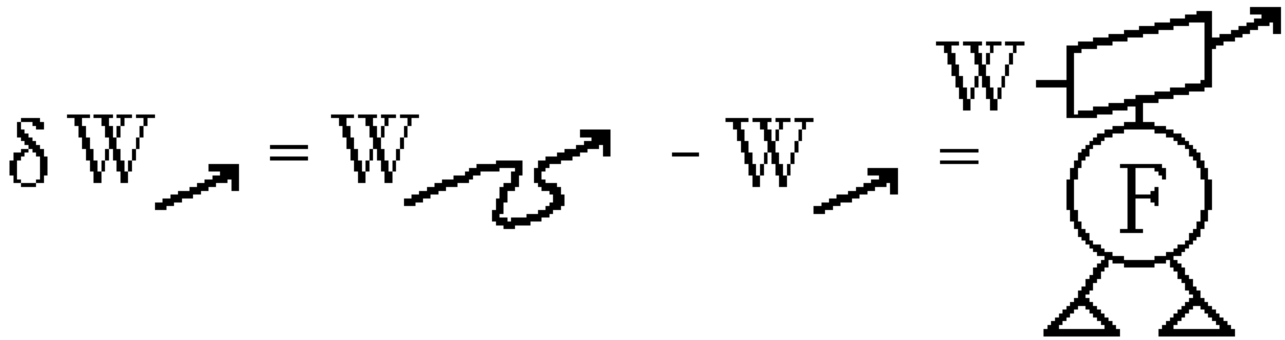
curvature tensor

Chern - Simons Lagrangian

Curvature is
$dA + A \wedge A$.

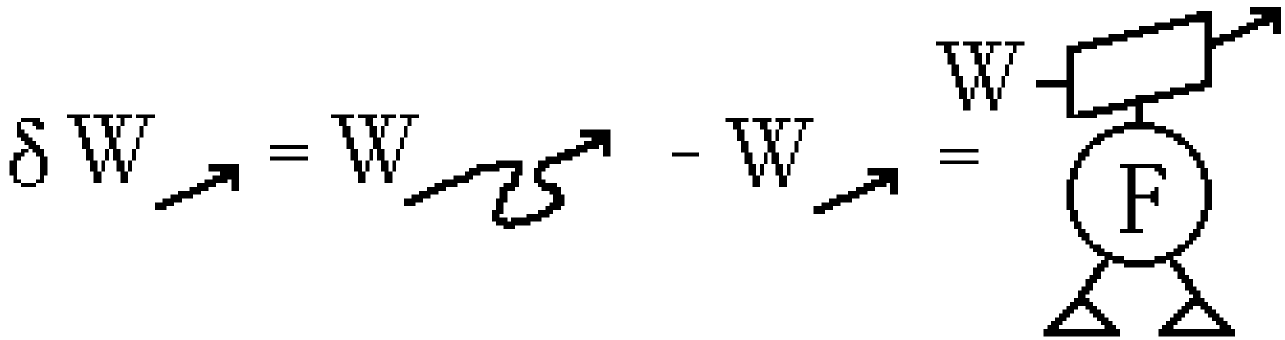
The Chern-Simons Lagrangian is
$L = A \wedge dA + (2/3) A \wedge A \wedge A$.

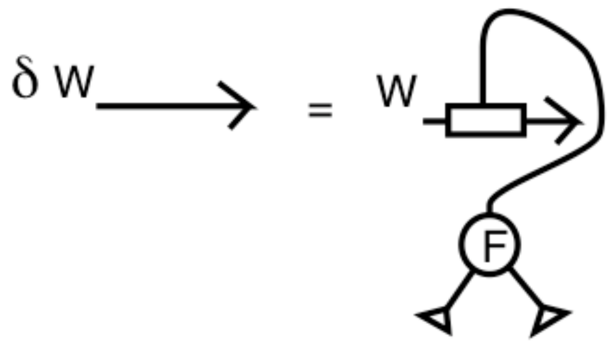
Differentiating L with respect to A
yields curvature.
(But you have to do it in detail to really see this.)



By an interesting calculation,
one finds that if you change the loop by a small amount,
then the Wilson loop changes by an insertion of
Lie algebra coupled with the curvature tensor.

This is just like classical differential geometry
where parallel translation around a small loop
measures curvature.

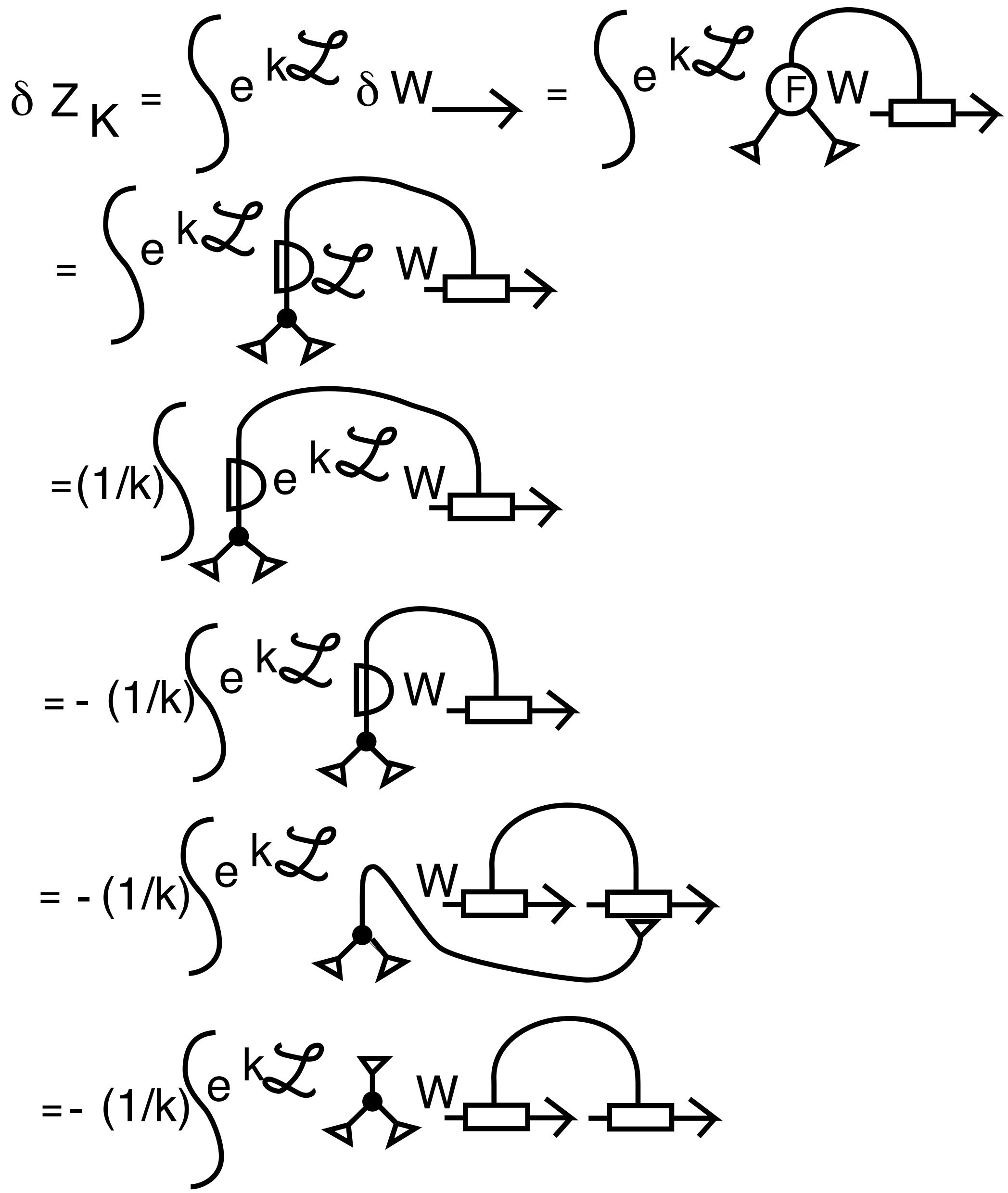




Curvature enters in when one evaluates the varying Wilson loop.

We can put all these facts together and find out how Witten's Integral behaves when we vary the loop.

The next slide tells this story in Diagrams.



δ Z
K
=
e
k
δ W
=
e
k
F
W
=
e
k
W
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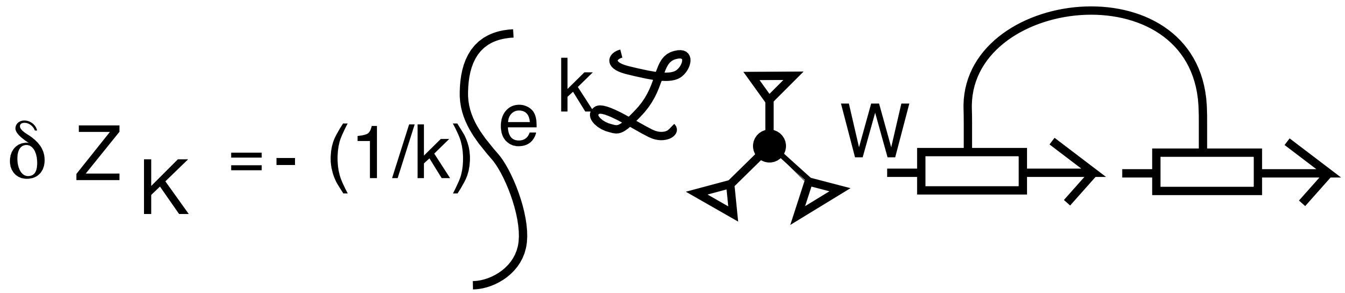
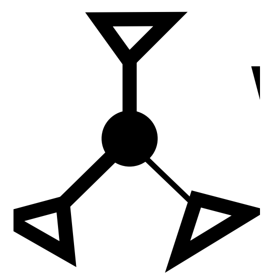


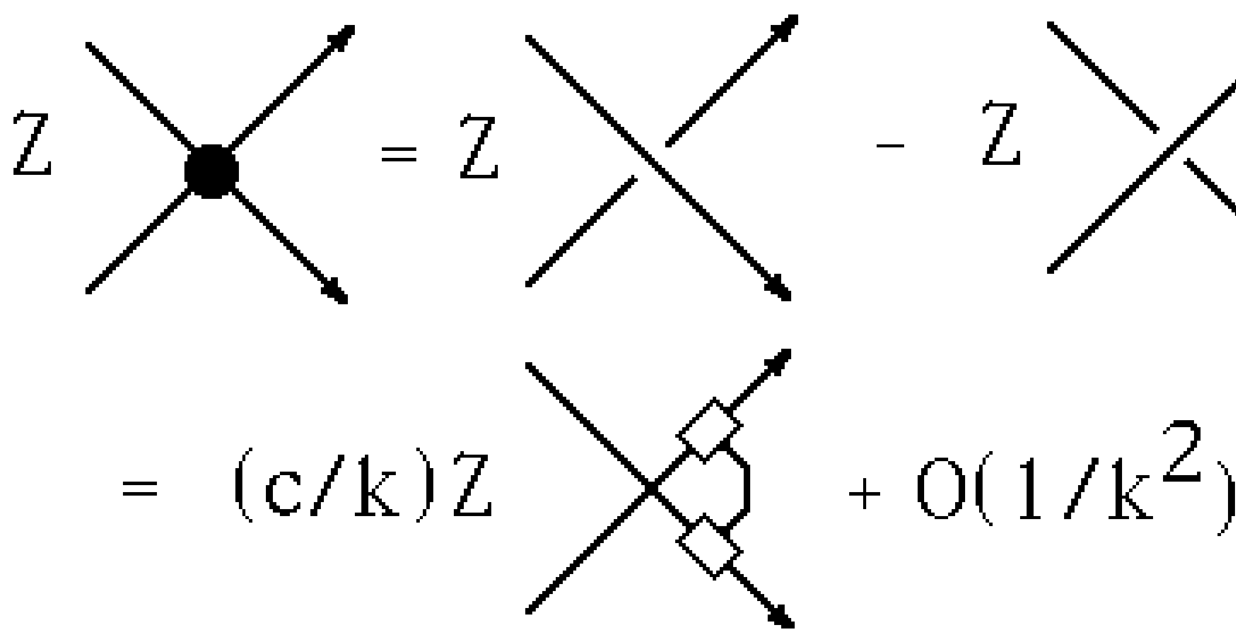
When you vary the loop,
Witten's integral changes by
the appearance of the volume form



and a double Lie algebra insertion.

There will be no change if the the
volume form is zero.
This can happen if the loop deformation
does not create volume.
That is the case for the
second and third Reidemeister moves
since they are "planar".

Hence we have shown (heuristically) that
Z_K is an invariant of "regular isotopy"
just like the bracket polynomial.



This is what happens when you switch crossings.
You get a "skein relation" involving Lie algebra insertions.

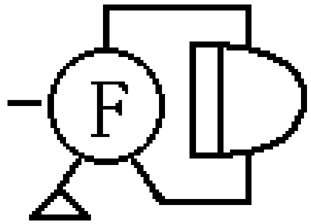
This formula leads directly to the subject of Vassiliev invariants, but we will not discuss that in this talk.

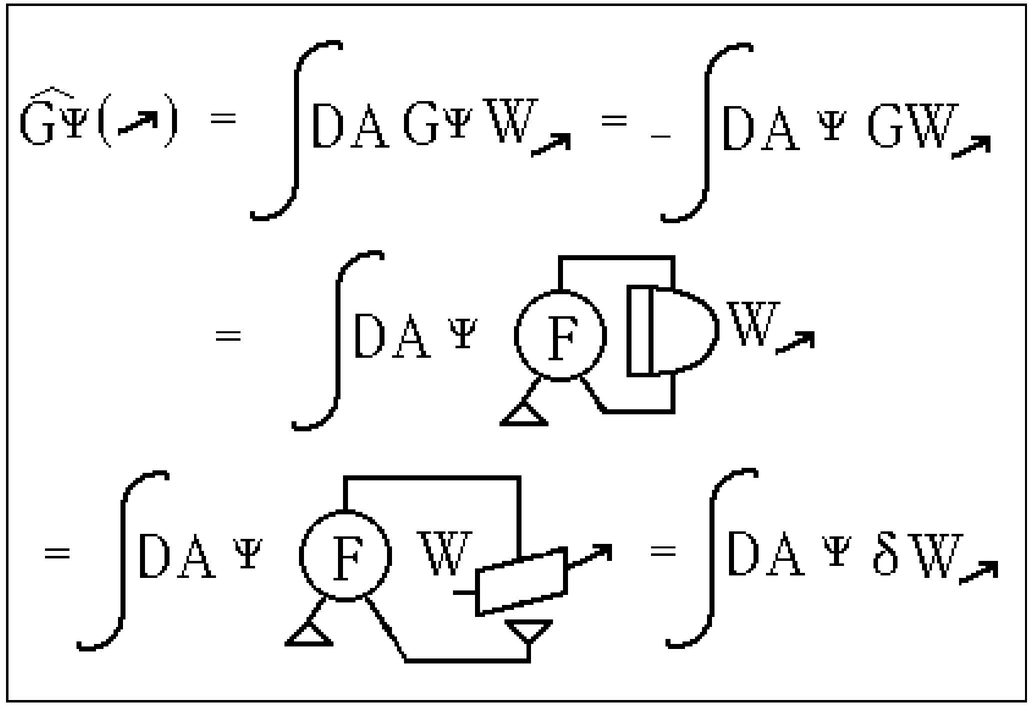
$$\hat{\Psi}(K) = \int DA\, \Psi(A)\, W_K$$

$$\widehat{\Delta\Psi}(K) = \int DA\, \Delta\Psi(A)\, W_K$$

$$= -\int DA\, \Psi(A)\, \Delta W_K$$

The Loop Transform: Start with a function defined on gauge fields. Integrate it against a Wilson loop and get a function defined on knots.
Transform differential operations from the category of functions on gauge fields to the category of functions on knots.

G = 

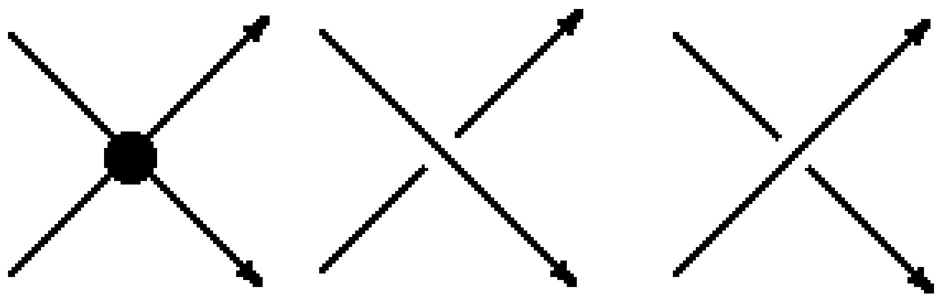


This differential operator occurs in the loop quantum gravity theory of Ashtekar, Rovelli and Smolin.

Its transform is the geometric variation of the loop!

The loop transform enabled
Ashtekar, Rovelli and Smolin to
see that the exponentiated
Chern-Simons Lagrangian could be seen
as a state of quantum gravity
and
that knots are fundamental
to this approach to a theory of
quantum gravity.

Knots, Links and Lie Algebras
Vassiliev Invariants

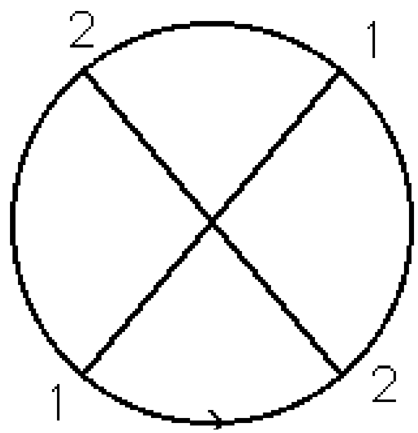


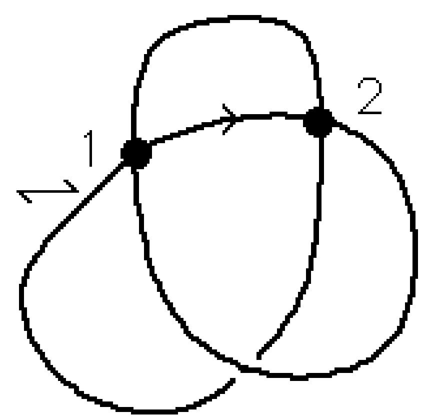
(K|*) (K|+) (K|-)

$$v(K|*) = v(K|+) - v(K|-)$$

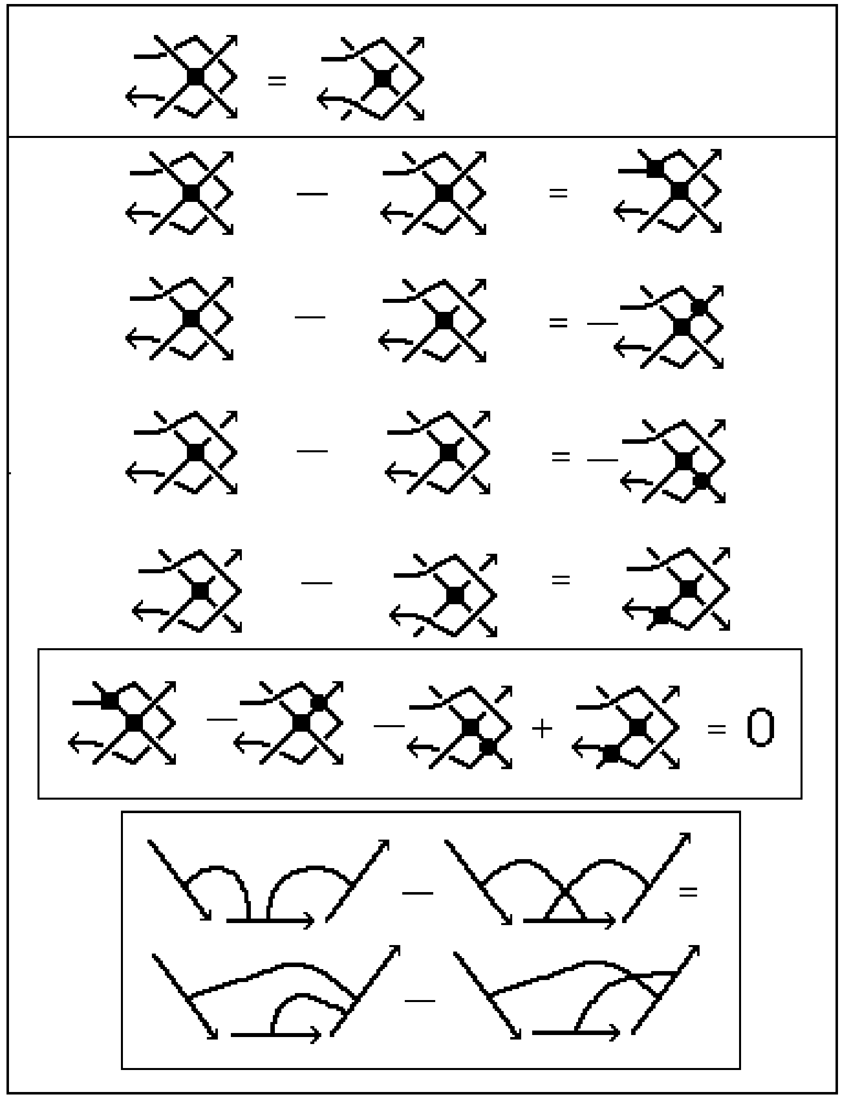
Skein Identity

Chord Diagram

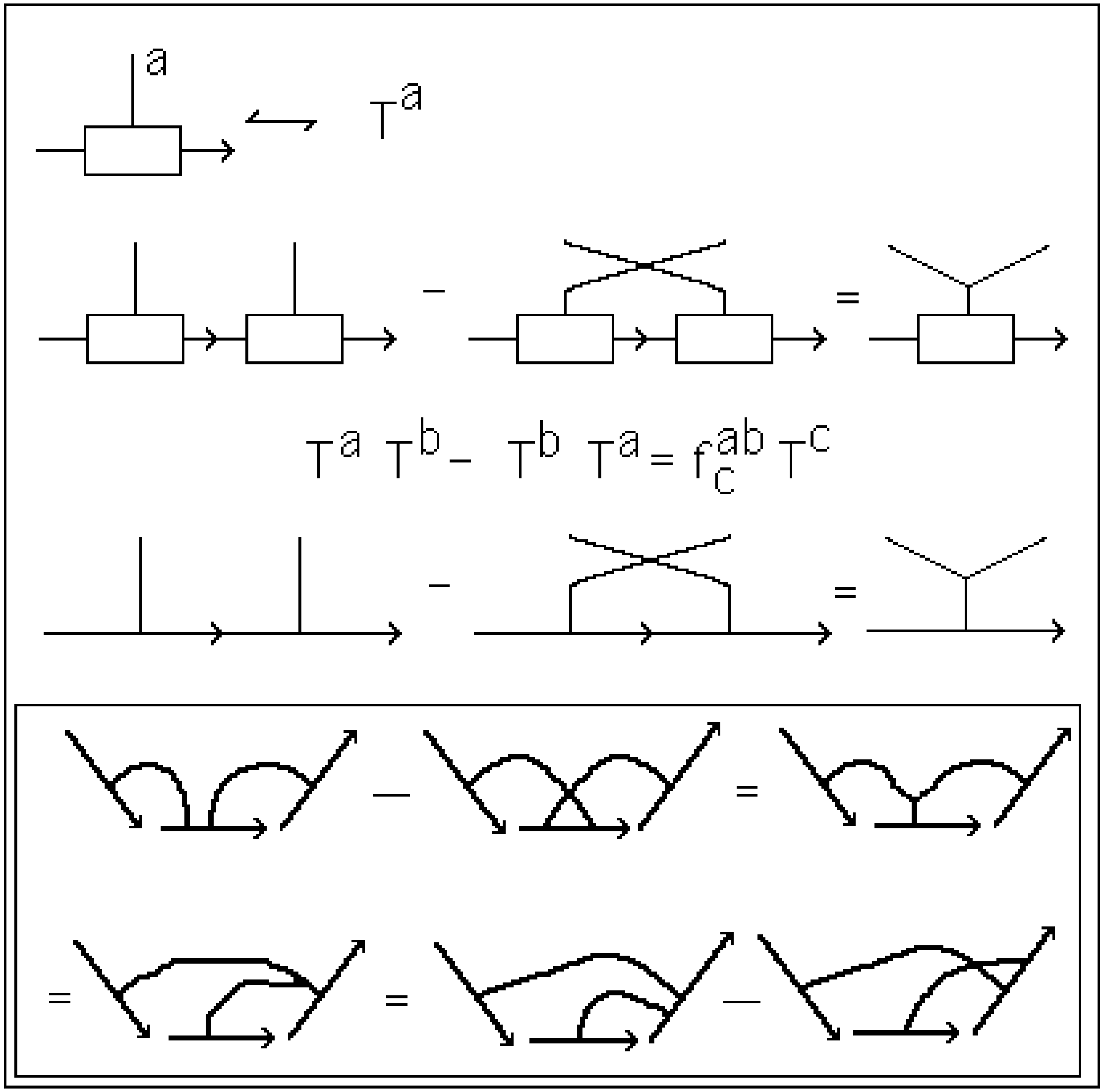




Four-Term Relation From Topology



Four Term Relation from Lie Algebra



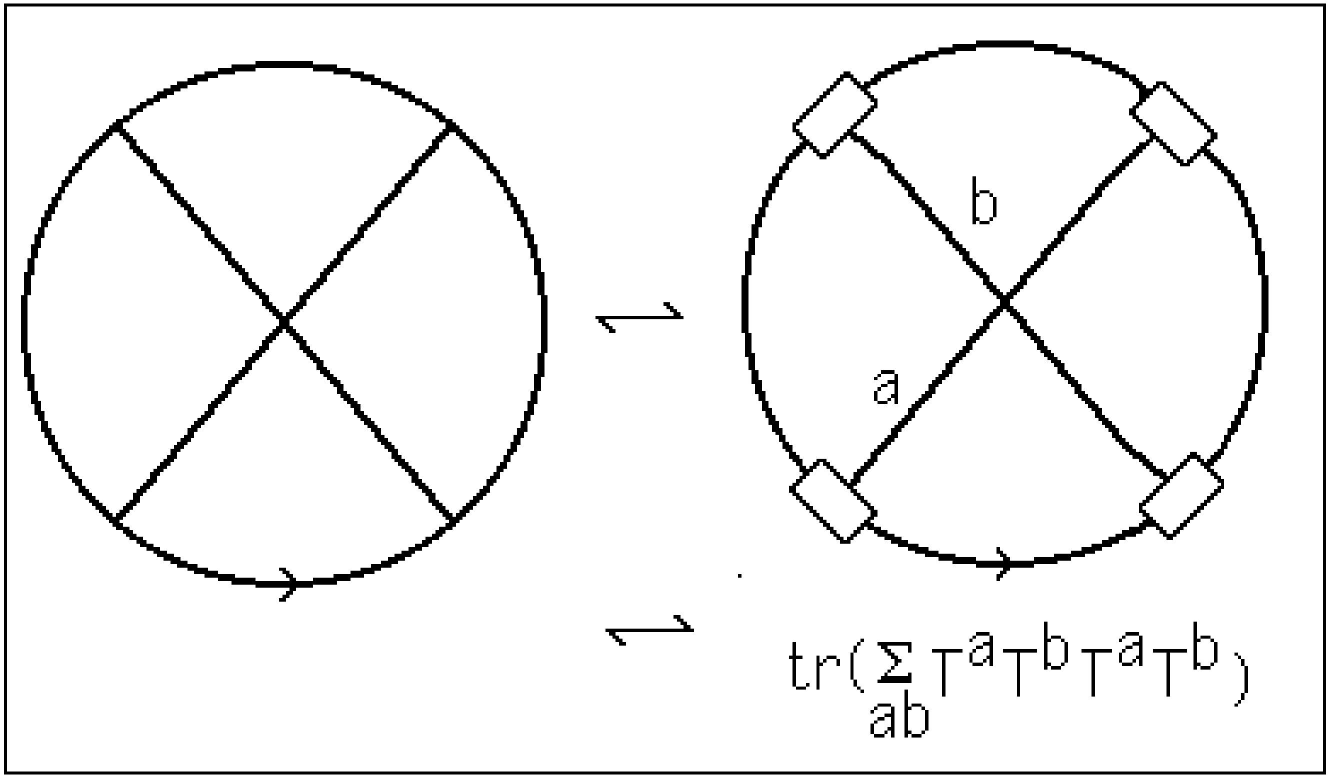
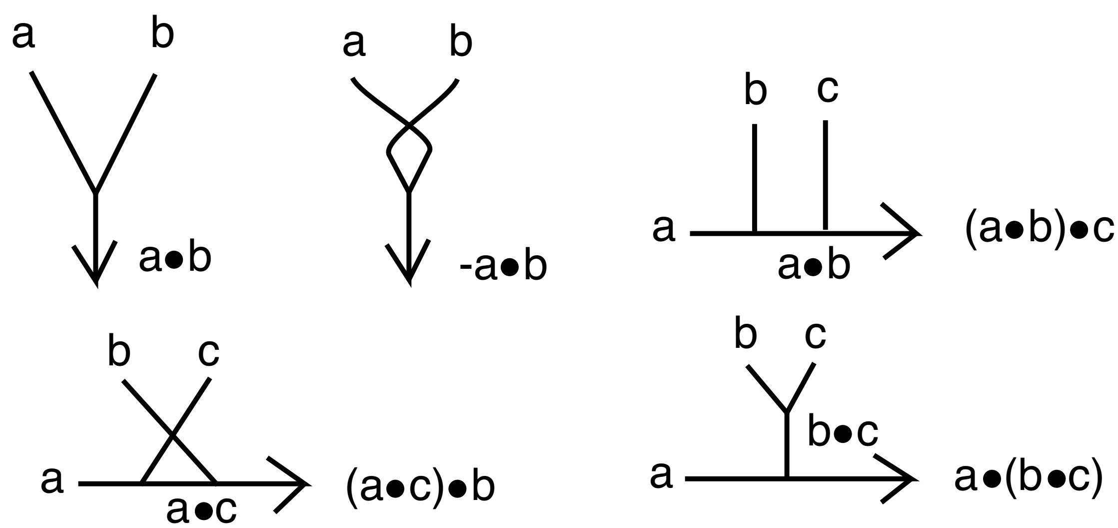
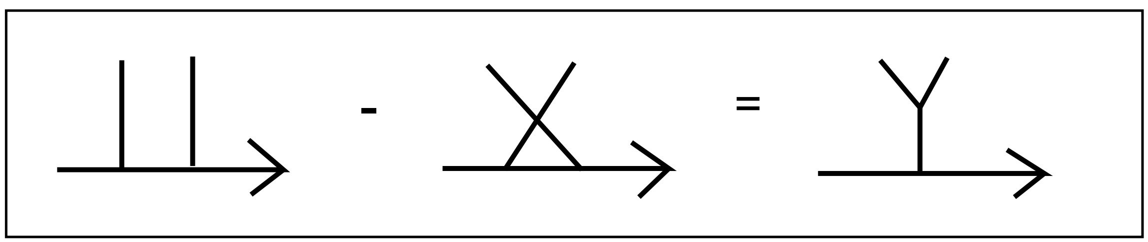


FIGURE 12. **Calculating Lie Algebra Weights.**

The Jacobi Identity





(a•b)•c - (a•c)•b = a•(b•c)
Hence
(a•b)•c + b•(a•c) = a•(b•c).

Lie algebras and Knots are linked
through the Jacobi Identity.

This is part of a mysterious
connection
whose roots we do not yet fully
understand.